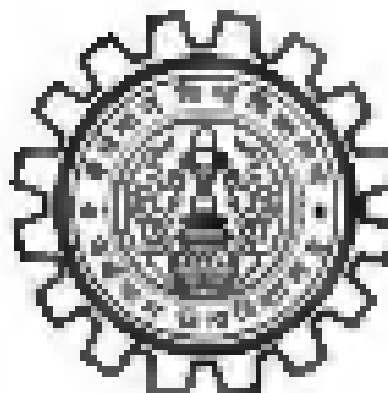


Study of black holes in standard and modified gravity

*Thesis submitted for the award of
Doctor of Philosophy in Science (Physics)
of
The University of Burdwan*



Submitted by

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(A.P.D.No./en/physics/43) dated 24.04.2021

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First, I want to thank my supervisor Dr. Anshu Rajawat, for all the time and the patience he/vided to me throughout all my research, giving. He was able to transform me the passion for research with experience and long discussions where fantastic ideas emerged. I think that I could not find a better person to guide me through this experience. He taught me how to deal with new physical problems. Then, I want to thank my family for supporting me throughout this journey. It is not an easy path as those who have already walked it know. But with the guidance of my supervisor and with the support of my family, the journey was made easy.

Sulhas Kumar Jha
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List of Publications

This is based on following works:

- 1. H. Jha, M. Karmakar, & Mahanta: JCAP04 (2017) 036
- 2. K. Jha, A. Rahaman: Eur. Phys. J. C 000 0000
- 3. H. Jha, S. Adhik, A. Rahaman: Eur. Phys. J. C 0000 0000
- 4. K. Jha, A. Rahaman: Eur. Phys. J. C 0000 0000
- 5. K. Jha, A. Rahaman: Eur. Phys. J. C 0000 0000
- 6. K. Jha, A. Rahaman: Eur. Phys. J. Part B, 00 (2017)

Additional Works are:

- 1. Rahaman, A. Rahaman and S.K.Jha: Modern Physics Letter A Vol. 26, No. 19, 2150007 (2011)
- 2. Adhik, S. K. Jha and A. Rahaman: Class. Grav. 34(2011) 2150004
- 3. Rahaman, A. Rahaman and S.K.Jha: Eur. Phys. J. Part B 0000000 (2017)

List of Symbols

Symbol	Description	Symbol	Description
c	velocity of light	D_{E}	tidal force
G	gravitational constant	h	Planck parameter
α, β	radial coordinate	α	derivative parameter of hairy Schwarzschild metric
r	areal radius	h_1	step in potential
r_+	event horizon radius	α_{E}	angle of deflection for strong gravitational lensing
λ	eff. parameter	α_{W}	angle of deflection for weak gravitational lensing
λ_{D}	wavelength in deuterium	α, β	lensing coefficients
λ_{D}	wavelength measured at infinity	θ_{E}	position of Einstein Einstein ring
α_{H}	angular velocity of black hole	Δ	separation of the image
T_{H}	Hawking temperature	α	specific angular momentum of black hole
β	charge parameter of black hole	M	mass of black hole
θ_{H}	non-rotational parameter	f	Lorentz violation parameter
λ	Planck parameter	V_{eff}	effective potential
λ_1	triplet parameter	R_{g}	size of the black hole
λ_2	derivative of non-rotational metric	λ	curvature property
λ	Planck constant	$r_{\text{ph}}, r_{\text{sh}}$	positions of event horizons
$r_{\text{ph}}, r_{\text{sh}}$	positions of Cauchy horizons	d_{L}	area of horizon
E_{ph}	energy of photon	dC	deviation from circularity
d_{L}	angular diameter	α_1	proper acceleration

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1. INTRODUCTION

Our present description of nature is based on two mutually successful theories: General Relativity (GR), a classical theory describing all gravitational phenomena, and the Standard Model (SM) of particle physics, which provides a quantum description of all other interactions. General relativity has emerged as a highly successful model of gravitation and cosmology, which has so far passed many nontrivial observational and experimental tests. However, there are strong indications that the theory is incomplete. The problem of quantum gravity and the question of the reality of quantum singularity remains open. Observational data that is taken as evidence for dark energy and dark matter could indicate the need for new physics. It is widely expected that these theories are merely the low-energy limit of some more fundamental theory that would take over as the characteristic energies involved in experiments approach the Planck scale, 10^{19} GeV. Experimental information to guide the development of a Planck-scale theory would, by conceptual thinking, come from Planck-scale experiments, which are likely to remain inaccessible for quite the future. An alternative approach is to search for small deviations from known physics (the SM and GR) in present-day experiments, with the hope that small deviations, if found, would provide information about the underlying theory. Lorentz symmetry, the idea that physical results are unchanged under rotations and boosts of the system, is the pillar of both the SM and GR. Gross violation of the symmetry, if found, would provide a novel signal of new physics. Moreover, the possibility of violation of this symmetry has been demonstrated in conditions for the underlying theory, like string. Here, we consider a special class of theory, called Einsteinian model, where Lorentz symmetry is spontaneously broken. The Lorentz violation arises from the dynamics of a single vector X_μ that assumes vacuum expectation value, \bar{X}_μ , which is assumed to zero. The vacuum value \bar{X}_μ acts as a fixed background field that spontaneously breaks Lorentz symmetry.

II. THEORY OF GENERAL RELATIVITY/EINSTEIN'S EQUATION

In 1905 Einstein published his special theory of relativity. While this theory has been able to predict many observed phenomena, it was not complete in its two ways. To explain matter and energy changing the geometry of spacetime, Einstein proposed a new theory. Shortly known as general theory relativity. This theory is characterized by a set of dynamic equations expressed as Einstein Field Equations showing how gravitational interaction, as a result of spacetime curvature is mass and energy. Albert Einstein published this new theory Gravity as a tensor equation (1915). Movement of the body under gravitational field is almost completely described by geodesic equation. Einstein used three principles in his development of general theory of relativity namely Weak Principle, Equivaly Principle and Cosmological principle. Einstein's field equations are given by

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \kappa T_{\mu\nu} \\ \Rightarrow G_{\mu\nu} &= \kappa T_{\mu\nu} \end{aligned} \quad (1)$$

Tensorial form of the above equation is

$$R_{\mu\nu} = \kappa(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \quad (2)$$

For $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ are the components of Einstein tensor, and T is defined by $T = g^{\mu\nu}T_{\mu\nu}$, κ being combination of metric tensor. If the speed of the light equal to $(-1, 1, 1, 1)$ then equation (1) becomes

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \kappa T_{\mu\nu} \\ \Rightarrow G_{\mu\nu} &= \kappa T_{\mu\nu} \end{aligned} \quad (3)$$

which takes tensorial form as

$$R_{\mu\nu} = \kappa(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \quad (4)$$

On solving Einstein's equation, one obtains the Schwarzschild metric as

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

where M is the mass of the black hole in unit of length and $G = c = 1$ for the units choice. Since the coefficient of the metric does not depend on t , and the metric does not have any cross term involving space-like and time-like increments, it is a static metric. The metric has two singularities, one at $r = 0$ and another at $r = 2M$. The first one cannot be removed using a change of coordinates, but the second one can be removed using a different choice of coordinates. It is called event horizon. Since the metric is static and spherically symmetric, E , the electric energy per unit mass of the particle and L , the angular momentum per unit mass, both with respect to an observer stationary at infinity, are constants of motion. Thus, we study circular motion of a particle around the black hole and the proper period of the particle in the circular orbit is found out to be

$$\tau = \frac{r^3}{2} + 2\pi \frac{r^2}{\sqrt{2M}} \sqrt{1 - \frac{12}{r}}. \quad (6)$$

Periodic orbit also studied for Schwarzschild black hole. The small oscillation, the scattering process through $\frac{1}{\sqrt{2M}}$ gravitational redshift and surface gravity for Schwarzschild black hole are also studied.

The Kerr solution, which describes a rotating black hole in 4D spacetime, is actually the solution of Einstein's equations in vacuum. In other words, the metric is solution of an empty rotating body. The Kerr metric, in Boyer-Lindquist coordinates reads [1]

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mr a \sin^2 \theta}{\rho^2} dt d\phi + \frac{r^2}{\Sigma} dr^2 + \rho^2 d\theta^2 + \frac{A \sin^2 \theta}{\rho^2} d\phi^2, \quad (7)$$

where $a = \frac{J}{M}$, J being the angular momentum of the black hole and

$$\begin{aligned} \rho^2 &= r^2 + a^2 \cos^2 \theta, \\ \Sigma &= r^2 - 2Mr + a^2, \\ A &= (\rho^2 + a^2)^2 - 2a^2 r \sin^2 \theta. \end{aligned} \quad (8)$$

The equation $\Sigma = 0$ has two real solutions when $a \leq M$. The larger of the two solutions gives the event horizon r_+ and the smaller one gives the Cauchy horizon r_- . The surface given by $\rho = 0$ is called naked limit surface if no particle, even photon, can remain static within this surface. The region between event horizon and the naked limit surface is called ergosphere. We also study surface gravity and Hawking temperature for the Kerr black hole. Penetrating which is unique for rotating black holes are also studied here. Black hole solutions for Kerr black hole are studied here.

In [35] Ben-Daniel a charged rotating BH solution which came to be known as the Kerr-New (KN) solution. The rotating charged metric in Boyer-Lindquist coordinates (t, r, θ, ϕ) is [3]

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mr a \sin^2 \theta}{\rho^2} dt d\phi + \frac{r^2}{\Sigma} dr^2 + \rho^2 d\theta^2 + \frac{A \sin^2 \theta}{\rho^2} d\phi^2, \quad (9)$$

where

$$\rho^2 = r^2 + a^2 + \frac{Q^2}{2} \sin^2 \theta, \quad \Sigma = r^2 + a^2 - 2Mr + \frac{Q^2}{2} \sin^2 \theta, \quad A = (\rho^2 + a^2)^2 - 2a^2 r \sin^2 \theta. \quad (10)$$

where $Q = Q/M^2$, Q being the charge of the black hole. Here we also study Hawking temperature and other effects for the Kerr-New black hole.

III. SPONTANEITY OF LORENTZ SYMMETRY BREAKING AND THE HUNGARIAN MODEL

General relativity and the standard model of particle physics are two very successful field theoretical models that enable us to describe our Universe. The formulation of both these theories is based on the well established Lorentz symmetry. However, the regime of applicability and the nature of various trends describing the Universe by these two theories are profoundly different. The general relativity describes the gravitational interaction and it is a classical field theory in the curved spacetime and there is no direct way to include the quantum effects. On the contrary, the standard model describes the other fundamental interactions and it is the quantum field theory in the flat spacetime that neglects all gravitational effects, but to study the physical system in the vicinity of the Planck scale [12–14] (GeV), the effect due to gravity cannot be ignored, since the gravitational interaction is strong enough in that energy scale. Therefore, the study of physics in the vicinity of the Planck scale necessarily needs the utilization of general relativity

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2. The second part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

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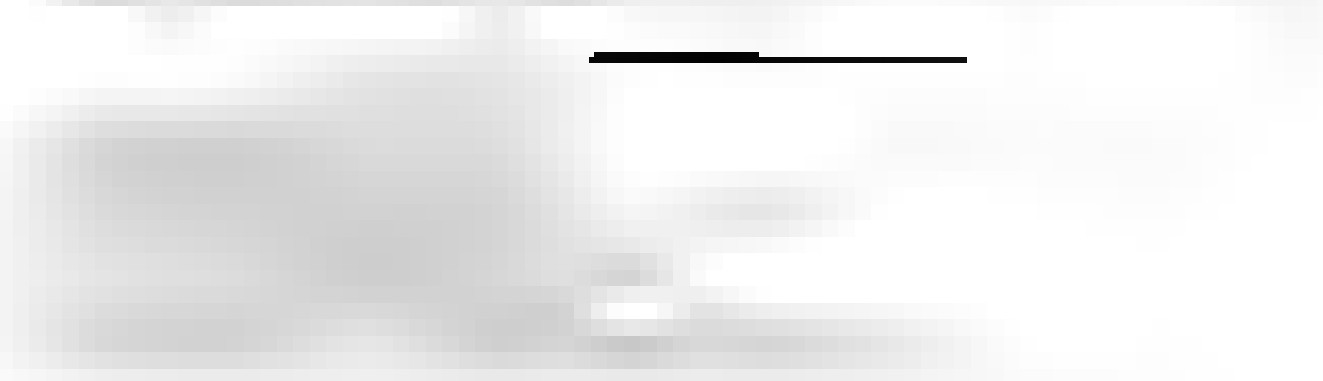
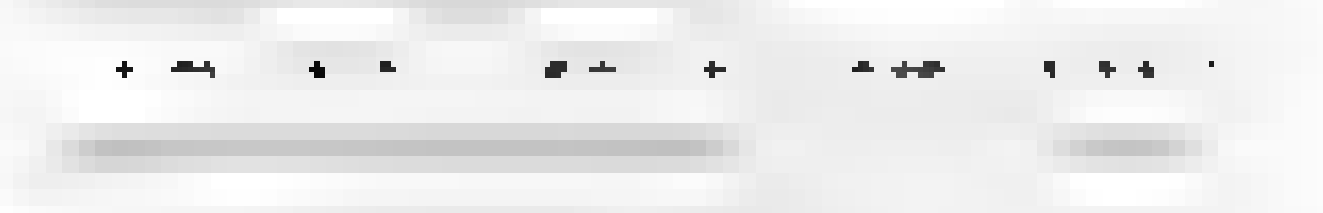
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7. The seventh part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

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8. The eighth part of the document is a list of the names of the persons who have been appointed to the various offices of the city.



Introduction

The first part of the paper discusses the importance of the problem and the need for a solution. It then presents a brief overview of the existing literature and the proposed approach. The second part of the paper describes the methodology used in the study, including the data collection and analysis techniques. The third part of the paper presents the results of the study, including the findings and conclusions. The fourth part of the paper discusses the implications of the findings and the need for further research. The fifth part of the paper provides a summary of the paper and the authors' contact information.

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Chapter 1

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Then, upon differentiating with

respect to r , S^2 with respect to r , we get

$$dS^2 = \frac{d^2}{2r} dr + \frac{d^2}{2r} dr. \quad (1.1)$$

Then, S^2 and components of energy-momentum tensor T_{ij} varying r with respect to r as obtain

$$dS^2 = \frac{d^2}{2r} dr + \frac{d^2}{2r} dr + R_{ij} \frac{dr}{2r} + \frac{dr}{2r} + \frac{dr}{2r} \quad (1.2)$$

Combining above two equations we get

$$dS^2 = \frac{d^2}{2r} (R_{ij} + \frac{dr}{2r} R_{ij}) + \frac{dr}{2r} + \frac{dr}{2r} \quad (1.3)$$

Adding in eqn. (1.1)

$$\frac{d^2}{2r} \frac{dr}{2r} = \frac{d^2}{2r} (R_{ij} + \frac{dr}{2r} R_{ij}) + \frac{dr}{2r} + \frac{dr}{2r} \quad (1.4)$$

According to the above principle

$$\frac{d^2}{2r} \frac{dr}{2r} = \frac{d^2}{2r} \frac{dr}{2r} \quad (1.5)$$

Then, we get

$$R_{ij} = \frac{d^2}{2r} R_{ij} + \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} \quad (1.6)$$

Eqn. (1.6) can be written as the following equation:

$$R_{ij} = \frac{d^2}{2r} R_{ij} + \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} \quad (1.7)$$

Then, $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is the Einstein tensor and is defined by the equation (1.7) as follows:

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} \quad (1.8)$$

where G_{ij} is defined as

$$G_{ij} = \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} + \frac{dr}{2r} R_{ij} \quad (1.9)$$

Equation (1.9) and (1.1) are known as Einstein's field equations.

1. Solutions of Einstein's field equation

Einstein's field equation is a set of partial differential equations. However, solutions of these equations are different solutions of Einstein's field equations for different geometrical structures. Let us first consider spherical symmetry. Einstein's field equation

1.1 Spherical symmetry

The first solution of Einstein's equation is a solution by Schwarzschild in 1916. He obtained the solution for vacuum gravitational field produced by a spherically symmetric mass. For such distribution of mass, general relativity gives the solution as

$$ds^2 = -\frac{dr^2}{1 - \frac{2M}{r}} - \frac{dr^2}{1 - \frac{2M}{r}} - \frac{dr^2}{1 - \frac{2M}{r}} - \frac{dr^2}{1 - \frac{2M}{r}} \quad (1.10)$$

and the second condition yields

$$M^2 = 1 - \frac{2M}{r} = 0 \quad (7.21)$$

The quadrupole moment Q is given by

$$Q = \frac{1}{2} \int r^2 \rho(r) dr \quad (7.22)$$

For $r = 1.737$ there is one solution. The solution corresponds to an unstable orbit and the positive sign corresponds to stable orbit. At $r^2 = 1.41^2$ we have the first or special orbit. The maximum for equality is also stable. In addition, at $r^2 = 0.94$ there is a quadrupole orbit, parallel to the orbit of the first orbit. The equation (7.22) can be written

$$Q = \frac{1}{2} \int \frac{r^2}{1 - \frac{2M}{r}} dr \quad (7.23)$$

We get the following expression of Q using equation (7.23) and (7.21)

$$Q = \frac{1}{2} \int \frac{r^2}{1 - \frac{2M}{r}} dr \quad (7.24)$$

The above calculations lead us to compute that the period of the orbit is $\frac{2\pi}{\omega}$ and the path is defined as

$$\frac{h}{L} = \frac{2}{r} = \frac{1}{r^2} \sqrt{1 - \frac{2M}{r}} \quad (7.25)$$

Thus, the proper period of the path is $\frac{2\pi}{\omega}$ and the period of the orbit is $\frac{2\pi}{\omega}$

$$\frac{1}{L} = \frac{1}{r^2} \sqrt{1 - \frac{2M}{r}} \quad (7.26)$$

7.2 Experimental test of general theory of relativity

Using the theory of the gravitational field and the h and c fields, the general theory of relativity is usually tested. With the theory of general relativity, in the weak field limit, general theory of relativity is usually approximated to the Newtonian gravity. Therefore, the observations of the general theory of relativity should not be included in the general theory of relativity. However, the observations of the general theory of relativity can explain those results reported by general theory of relativity.

7.2.1 The gravitational field of relativity

Using the differential equation of the field of the field, the equation of the field is given by

$$\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \right) = 0 \quad (7.27)$$

where $\frac{\partial}{\partial t} = \frac{\partial}{\partial r}$ is the general theory of the above equation is

$$\frac{\partial}{\partial t} \left(\frac{1}{r} \right) = \frac{1}{r} \quad (7.28)$$

Then $\frac{\partial}{\partial t} = \frac{1}{r}$ and $\frac{\partial}{\partial r} = \frac{1}{r}$ is the general theory of the field. This is usually considered by the general theory of relativity.

1.2.3 Bending of light

One of the upcoming results given by general theory of relativity is the bending of light by gravity. In the absence of mass, light rays travel in a straight line. But in the presence of mass, as for example in stars and planets, light rays coming from distant stars are deflected in such a way that their direction is deflected through deflection angle δ , which is directly proportional to the closest distance r_0 of them to the mass. As a result of this deflection, distant stars and galaxies are visible. It is this phenomenon that is used in cosmological lensing process. Cosmological lensing process has in the past helped to find objects that were thought to be too bright and closer objects.

1.2.4 Radar echo delay

Radar echo delay is one of the classical tests of general relativity. To explain it, we consider two planets A and B with masses M_A and M_B in between them. Now, if we send a radar signal from planet A to B then in the absence of the signal it would travel in a straight line and hence, it would take the time to return back to A . But in the presence of the bodies A and B , due to gravitational time dilation, the signal will take longer time to return back to planet A after getting reflected from the planet B . This delay in reaching the signal due to gravitational field of the masses is known as radar echo delay.

1.2.5 Gravitational Redshift

Now, we consider the case of Earth's the proper time to the background field is given by

$$ds^2 = \frac{1}{1 - 2M/r} dt^2 - dr^2 \quad (7.21)$$

Since the wavelength of radiation is proportional to the period of oscillation, we can write

$$\lambda = \lambda_0 \sqrt{1 - 2M/r} \quad (7.22)$$

for the observer between the two heights r_1 of radiation emitted r_1 and the wavelength λ_2 received at infinity. The redshift z is defined by

$$z = \frac{\lambda_2 - \lambda_1}{\lambda_1} \quad (7.23)$$

and we get

$$z = \frac{1}{\sqrt{1 - 2M/r_1}} - 1 \quad (7.24)$$

1.2.6 Proper acceleration and surface gravity

From the standard definition, the proper acceleration is given by

$$a^\mu = \frac{d^2 x^\mu}{d\tau^2} \quad (7.25)$$

where the acceleration a^μ is defined by

$$a^\mu = \frac{dx^\mu}{d\tau} = \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} \quad (7.26)$$

The components of the 4-velocity of a something observer are given by

$$u^\mu = (1, 0, 0, 0) \quad 0 \leq t \leq 2\pi \quad (7.27)$$

The non-zero components of the 4-acceleration is given by

$$a^\mu = \left(\frac{M}{r^2}, 0, 0, 0 \right) \quad (7.28)$$

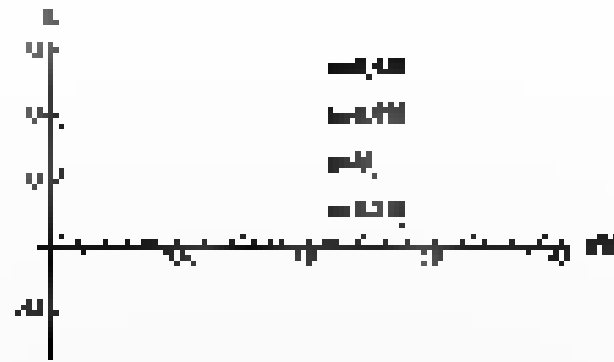


Figure 1: The function $f(x)$ versus x from Eq. (1) with $\gamma = 0.5$ (see eq. (1) and eq. (2)).

We use the same setup, that the equation is a linear combination of the δ and δ' terms, with the δ term being $\delta(x) = \gamma$.

3.1. Static limit surface and ingoing wave

We describe here the ingoing wave at the static limit, described by $\gamma = 0$. For a stationary particle, we have $\dot{x} = 0$, $\dot{p} = 0$, constant. Thus

$$\left(\frac{d}{dt}\right)^2 = 0. \quad (3)$$

For $\gamma = 0$, the static condition must be satisfied and hence, within the surface ρ_{max} , the static particle is the stationary state of the deformed $\rho_{\text{max}} = 0$ at the static limit surface.

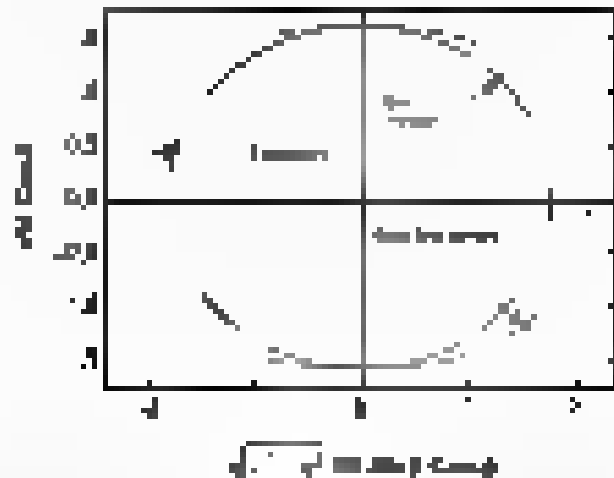


Figure 2: Static limit surface ingoing wave and static limit surface. See text for details.

The radius of the static limit surface is given by

$$r_s = R + \sqrt{R^2 - \gamma^2} \quad (4)$$

The static limit surface r_s marks the γ condition where a peak within the surface exists. The region between the static limit surface and the static horizon is called ingoing wave. Since ingoing wave exists in the region, it is possible to extract energy from the ingoing wave through Penrose process [12]. In the ingoing wave, we show existence of ingoing wave with $\gamma = 0$ plus the area of the ingoing wave is zero $\gamma = 0$.

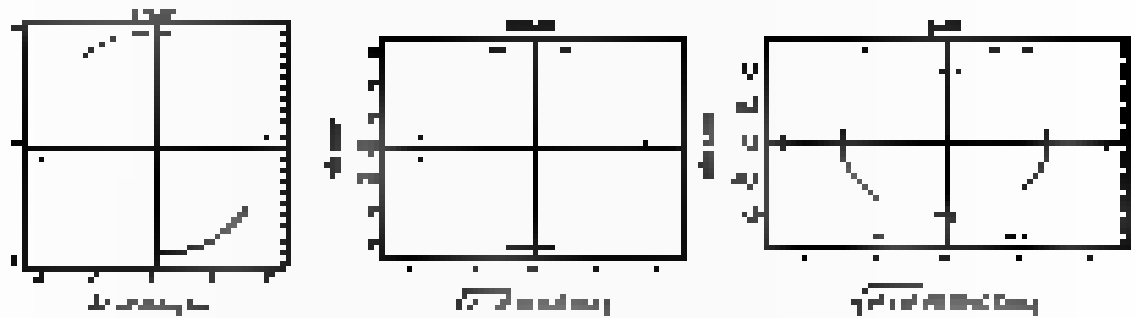


Figure 3 Variation in magnetic field.

4.3 Surface gravity and Hawking temperature

The metric given by (24)

$$= -\frac{dr^2}{2\pi} + \frac{r^2}{2\pi} dt^2 \quad (7.34)$$

where $K = \frac{2\pi}{r^2}$ is the surface gravity and $\frac{r^2}{2\pi}$ is the temperature, we get

$$\frac{dr^2}{2\pi} + \frac{r^2}{2\pi} dt^2 = \frac{r^2}{2\pi} dt^2 \quad (7.35)$$

The surface gravity can be obtained from (24) using the relation

$$T_H = \frac{1}{2\pi} \quad (7.36)$$

Thus, the Hawking temperature is

$$\frac{r^2}{2\pi} dt^2 \quad (7.37)$$

4.4 Coordinate system and differential equations of motion

Consider two sets of coordinates (t, r) and (τ, ρ) in the coordinate system (t, r) related to the coordinate system (τ, ρ) . We assume by the metric, coordinate system of reference. For the energy equation, we have

$$\frac{d\tau}{d\rho} = \frac{r^2}{2\pi} \quad (7.38)$$

Substituting the metric coefficients, we get

$$\frac{d\tau}{d\rho} = \frac{r^2}{2\pi} \left(\frac{dr}{d\rho} \right)^2 + \frac{r^2}{2\pi} \left(\frac{dt}{d\rho} \right)^2 \quad (7.39)$$

where $\frac{dr}{d\rho}$ is the coordinate of the angular momentum of the particle normal to the equatorial plane, $\frac{dt}{d\rho}$ is the coordinate of the coordinate of the particle, r is the radius of the black hole and ρ is the coordinate of the particle, and K is the energy of the particle per unit mass.

For the angular momentum equation, we have

$$\frac{d\tau}{d\rho} = \frac{r^2}{2\pi} \quad (7.40)$$

from which we obtain

$$\frac{d\tau}{d\rho} = \frac{r^2}{2\pi} \left(\frac{dr}{d\rho} \right)^2 + \frac{r^2}{2\pi} \left(\frac{dt}{d\rho} \right)^2 \quad (7.41)$$

Answer: results of the static radial motion are by Cartesian coordinates the usual Laplace-Dirichlet boundary problem. In order to be sure that the parameters in the Laplace problem are correct, the Hamiltonian representing the radial problem is given by

$$\frac{H^2}{2} = -\frac{d^2 R}{d\tau^2} \quad (2.50)$$

where τ denotes the proper time along the geodesic. With $\tau = 0$ the R coordinate is given

$$\frac{d^2 R}{d\tau^2} = -\frac{1}{R^2} + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 = -\frac{1}{R^2} + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.51)$$

$$+ \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.52)$$

From the boundary conditions that the radial coordinate must satisfy, the solution of the last equation of the form

$$R(\tau) = E + A_1 \tau + A_2 \tau^2 + A_3 \tau^3 \quad (2.53)$$

where E and A_i are functions only of the variable specified. For the choice (2.53) of R , we can write

$$\frac{d^2 R}{d\tau^2} = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.54)$$

$$\frac{d^2 R}{d\tau^2} = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.55)$$

The above equation yields

$$\frac{d^2 R}{d\tau^2} = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.56)$$

and

$$\frac{d^2 R}{d\tau^2} = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 + \frac{1}{2} \left(\frac{dR}{d\tau} \right)^2 \quad (2.57)$$

where Q is a parameter constant usually called the Q -parameter. There are additional two conserved parameters

$$\frac{L}{R} \quad \text{and} \quad \frac{E}{R} \quad (2.58)$$

Equations (1.54) and (1.55) along with equations (1.56) and (1.57) in terms of above conserved quantities can be reduced to

$$\begin{aligned} \frac{dR}{d\tau} &= + \frac{R}{2} \quad \frac{dR}{d\tau} = - \frac{R}{2} \\ \text{or} \quad \frac{dR}{d\tau} &= \pm \frac{R}{2} \\ \text{or} \quad \frac{dR}{d\tau} &= \pm \frac{R}{2} \end{aligned} \quad (2.59)$$

where \pm is a free parameter and

$$R(\tau) = R_0 e^{\pm \frac{\tau}{2}} \quad (2.60)$$

11.3 Frame-dragging

Let us consider frame-dragging effect in a region between 2 rotating black holes. For a particle falling from infinity, with $L = 0$, angular momentum is conserved

$$\sqrt{g} \frac{d\phi}{dt} = \frac{[1 - \frac{2M}{r} - \frac{J^2}{r^2 \sin^2 \theta}]^{1/2} \sin^2 \theta}{2} \cdot \Omega \quad (11.3)$$

and the equation (11.7) becomes

$$\sqrt{\frac{dt}{dr}} = \frac{2M}{r} \cdot \Omega \quad (11.4)$$

The angular velocity of the particle, as seen by a distant observer, is

$$\omega(r, \theta) = \frac{d\phi}{dt} = \frac{r \Omega}{[1 - \frac{2M}{r} - \frac{J^2}{r^2 \sin^2 \theta}]^{1/2}} \quad (11.5)$$

where ϕ is the azimuthal coordinate in Schwarzschild coordinates. Although the particles are falling radially with $\theta = \pi/2$, they acquire an azimuthal angular velocity during infall. From equation (11.5) we observe that the particle obtains angular velocity at the distance of the spin of the black hole. One can be tempted to identify frame-dragging caused by the real matter from Ω the rotating hole. The angular velocity of the hole, Ω_h , given by equation (11.5) at $r = \infty$ is the same as that of the rotating hole.

$$\Omega_h = \frac{\Omega}{32\pi} \quad (11.6)$$

11.4 Effective potential, orbital radius and conserved quantities

The radial equation at test $\mu = 1$ is given by substituting in the form

$$\sqrt{\frac{dr}{dt}} = \dots \quad (11.7)$$

The effective potential V_{eff} is

$$V_{eff} = [1 - \frac{2M}{r} - \frac{J^2}{r^2 \sin^2 \theta}]^{1/2} \sin^2 \theta \quad (11.8)$$

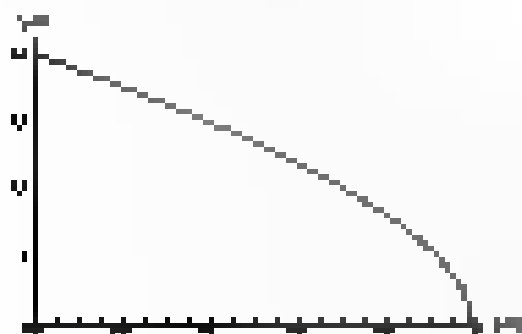
Note that $V_{eff}(r) \rightarrow 0$ and $V_{eff}(\infty) \rightarrow 0$. On the r - θ plane, the unstable spherical orbit is given by the following equation

$$r = \frac{2M}{\sin^2 \theta} \quad \text{or} \quad \frac{dr}{dr} = \frac{dV_{eff}}{dr} = \frac{dV_{eff}}{d\theta} = 0 \quad (11.9)$$

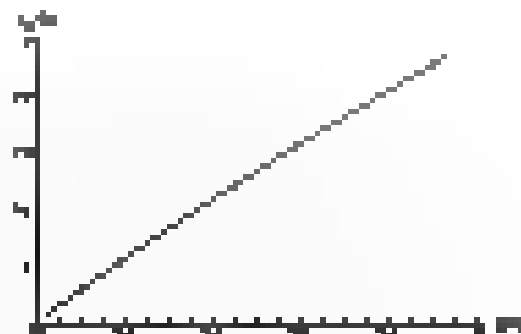
where θ is the value of the unstable orbit is

$$\Omega(1 + \cos^2 \theta) = \Omega_h (1 + \frac{J^2}{M^2 \sin^2 \theta}) \quad (11.10)$$

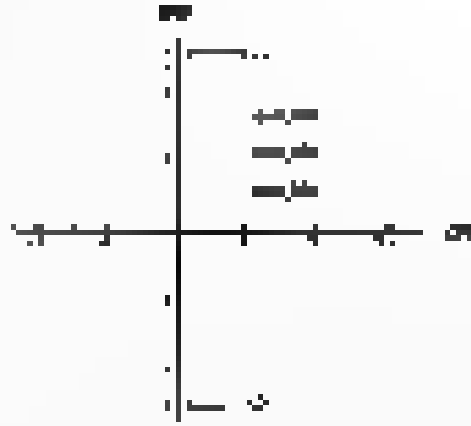
where the square root corresponds to retrograde orbits and the linear root corresponds to prograde orbits. In usual prograde orbits, particles rotate about the axis of symmetry of the system away from the black hole and in case of retrograde orbit, particles rotate about the axis of symmetry of the system away from the black hole.



(a) Variation of radius of prograde orbit



(b) Variation of radius of retrograde orbit

Figure 1: Phase plane plot of the system (1) in the (ϕ, ψ) plane.

As we can see from the plot, the system shifts from left to right as ψ increases.

1.4 Kerr-Sen black hole

In [21], a four-dimensional metric system that describes a rotating and electrically charged black hole in the Einstein-Maxwell theory. The black hole is given by the following metric and electromagnetic field:

$$ds^2 = -f_{\text{KS}} dt^2 + \frac{1}{f_{\text{KS}}} d\phi^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad (2.24)$$

where t is the time coordinate and ϕ, θ, φ are the angular coordinates. The metric is given by

$$f_{\text{KS}} = \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_-^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_0^2} - \frac{a^2}{r^2} \right), \quad (2.25)$$

where r_+ and r_- are the roots of the polynomial equation

$$r^2 - 2Mr + a^2 = 0, \quad (2.26)$$

where M is the mass of the black hole and a is the angular momentum. The electromagnetic field is given by the vector potential

$$A = \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_-^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_0^2} - \frac{a^2}{r^2} \right) dt + \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_-^2} - \frac{a^2}{r^2} \right) d\phi + \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) d\theta + \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) \sin^2 \theta d\varphi. \quad (2.27)$$

where

$$r_+ = \frac{1}{2} \left(2M + \sqrt{4M^2 - a^2} \right), \quad r_- = \frac{1}{2} \left(2M - \sqrt{4M^2 - a^2} \right), \quad r_0 = \frac{1}{2} \left(2M + \sqrt{4M^2 - a^2} \right). \quad (2.28)$$

From (2.24) and (2.25) we have

$$g_{\phi\phi} = \frac{1}{f_{\text{KS}}}, \quad g_{\theta\theta} = r^2, \quad g_{\varphi\varphi} = r^2 \sin^2 \theta, \quad g_{t\phi} = \frac{1}{4} \left(\frac{r^2}{r_+^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_-^2} - \frac{a^2}{r^2} \right) \left(\frac{r^2}{r_0^2} - \frac{a^2}{r^2} \right). \quad (2.29)$$

The metric (2.24) represents a four-dimensional black hole solution with a rotating and electrically charged black hole. The metric is given by

6.4.3 Even between and edge phases

The criticalities and the stability conditions are given by the solution of $\Delta = 0$ with $\delta = 0$

$$\Delta = 3\beta - \frac{1}{2} \pm \frac{\sqrt{1 - 6 - 4\beta^2 - 12\beta^3}}{2} \quad (6.14)$$

and the location of the outer limit surface is

$$\Delta = \Delta_{\pm} = \frac{1}{2} \pm \frac{\sqrt{1 - 6 - 4\beta^2 - 12\beta^3 - 12\beta^3}}{2} \quad (6.15)$$

The variation of size and shape of co-graphs of the K3-free black hole is shown below.

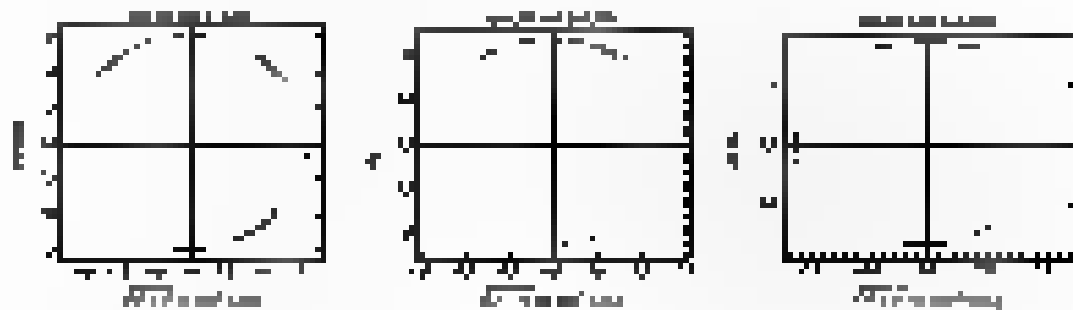


Figure 1: Evolution of edge co-graphs in size.

The size of the co-graphs we see can change from above, increasing with the number of black holes of total edge n

$$\Delta = 12\beta - \Delta \quad (6.16)$$

The parameter space for the K3-free black hole is shown below

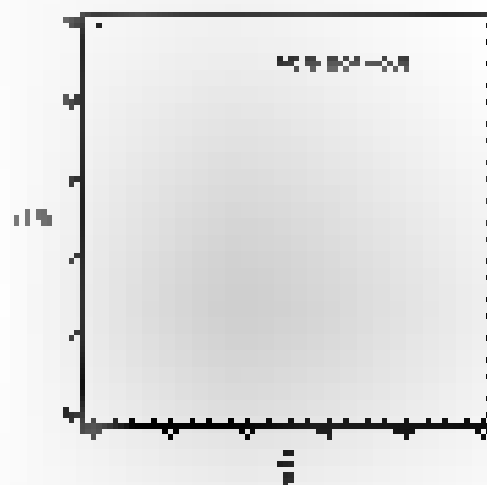


Figure 2: Parameter space for the K3-free black hole

The blue region in white we will get black hole and the white region in white (K3-free) will be black hole. The resolution is 256. The computation is performed. Can you black hole be found (shape and size) from graphically

3.3 Surface gravity and Hawking temperature

The surface gravity of the Kerr-Newman black hole, using equation (14), is given by

$$\kappa = \frac{\sqrt{2M - r_+^2}}{2M - 2r_+ + \sqrt{2M - r_+^2} - \frac{Q^2}{4M}}. \quad (3.11)$$

Then the Hawking temperature, using the relation

$$T_H = \frac{\kappa}{2\pi}, \quad (3.12)$$

is given by

$$T_H = \frac{1}{4\pi M} \frac{\sqrt{2M - r_+^2}}{2M - 2r_+ + \sqrt{2M - r_+^2} - \frac{Q^2}{4M}}. \quad (3.13)$$

3.4 Field equations

Following the same procedure as that used to derive the equations for radii production in case of Kerr-Newman black hole, one gets

$$\begin{aligned} \frac{dr}{dt} &= \pm \sqrt{E} - \sqrt{\frac{M}{2t}} - \frac{Q^2}{8t} \\ &= 2\sqrt{t} \left(\frac{dr}{dt} \right) = A - \frac{1}{4M\sqrt{t}} \\ &= 2\sqrt{t} \left(\frac{dr}{dt} \right) = 2M\sqrt{t} - \frac{1}{4M\sqrt{t}} - \sqrt{t} - 2M\sqrt{t} \end{aligned} \quad (3.14)$$

where A is an integration constant and

$$A(t) = (r^2 + (2 + r^2 - Q^2)^{1/2} - 2) + (1 - Q^2 - 8M) + 2(r + 1 - 2^2)r^2. \quad (3.15)$$

3.4.1 Spherical shell and black hole formation

On the other hand, the formation of spherical black hole given by the equation (3.14). For convenience, we have $r = 0$ and $r = 0$. The study about the behavior of Eq. (3.14) gives the information about the formation and production of the spherical black hole. For the Kerr-Newman black hole, one gets

$$\begin{aligned} &= \frac{1}{4M\sqrt{t}} \left(\frac{dr}{dt} \right) = \frac{1}{4M\sqrt{t}} \left(\frac{dr}{dt} + \frac{2M}{\sqrt{t}} + \frac{1}{4M\sqrt{t}} - \frac{1}{4M\sqrt{t}} \right) \\ &= \frac{1}{4M\sqrt{t}} \left(\frac{dr}{dt} + \frac{2M}{\sqrt{t}} + \frac{1}{4M\sqrt{t}} - \frac{1}{4M\sqrt{t}} \right) \end{aligned}$$

The new equation is given by

$$\begin{aligned} \frac{dr}{dt} &= \frac{1}{4M\sqrt{t}} \\ \frac{dr}{dt} &= \frac{1}{4M\sqrt{t}} + \frac{1}{4M\sqrt{t}} - \frac{1}{4M\sqrt{t}} \end{aligned} \quad (3.16)$$

By substituting the equation (3.16) into the equation (3.14),

and Kar3-Gem3 test factors. For Kar3-Gem3 test, we have taken $\alpha = 0.01$. We have considered representative $\beta = 0.1, 0.5$ and 1 for both of them.

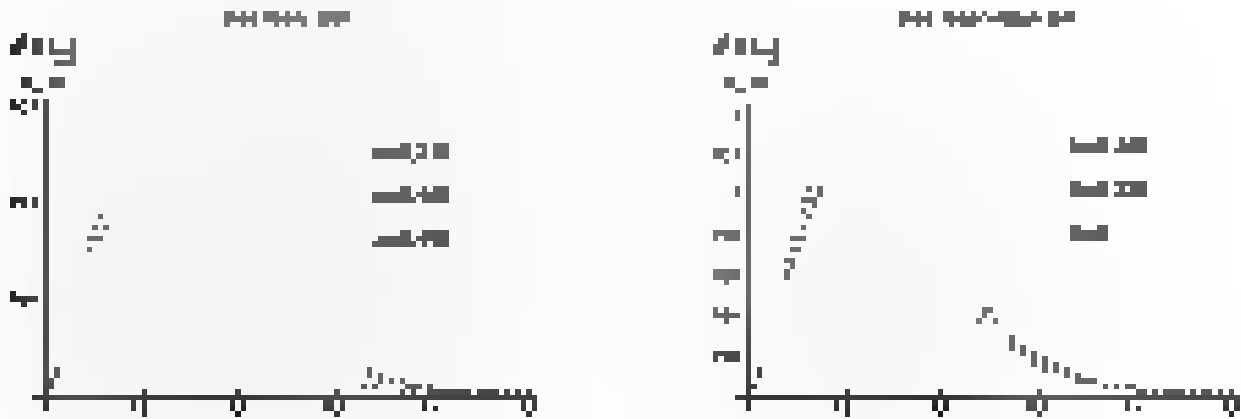


Figure 15: The figure on the left side shows the variation of test power with α for Kar3-Gem3 test with $\beta = 0.1, 0.5$ and 1 . The figure on the right side shows the variation of test power with α for Kar3-Gem3-Gem3 test with $\beta = 0.1, 0.5$ and 1 .

It is evident from above plots that the rate of increase in power with an increase in α whenever decreases with an increase in β .

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In this situation, Einstein equations take explicit generalized form

$$\begin{aligned} R_{\mu\nu} &= (1_{\mu\nu} - \gamma_{\mu\nu})\gamma_{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \gamma_{\mu\nu} R_{\alpha\beta} \gamma^{\alpha\beta} + \gamma_{\mu\nu} R_{\alpha\beta} \gamma^{\alpha\beta} \\ &= \frac{2}{3} \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \frac{2}{3} \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} \\ &= R_{\mu\nu} = 0 \end{aligned} \quad (3.13)$$

with

$$\begin{aligned} \gamma_{\mu\nu} &= R_{\mu\nu} - \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} \\ &= \frac{2}{3} \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} + \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} \end{aligned} \quad (3.14)$$

2.3 Schwarzschild-like Lense-Thirring rotating black hole solution

In this chapter we discuss some of the relevant features of the Schwarzschild-like black hole solution, which is usually (implicitly) assumed to be stationary of the modified Einstein equations (3.13). The solution has obtained by means of (3.13) by assuming that the spacetime is governed by the metric of the form

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3.15)$$

where r and θ are functions of M and a . The metric (3.15) is stationary, the time coordinate t is constant, which means the coordinate t is constant. With these assumptions, the static, spherically symmetric vacuum solution of modified equations (3.13) is found out to be [14]

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3.16)$$

where the parameter a is defined by $a = (M^2 - 4a^2)^{-1/2}$ and is called the Lense-Thirring parameter. In the limit $a \rightarrow 0$, the metric (3.16) yields the well known Schwarzschild solution. Although the metric has a structural similarity with the Schwarzschild metric, it is known to be very different from this, as the solution satisfies the following Schwarzschild-like

$$R_{\mu\nu} = \frac{1}{(1 + 4a^2)^2} \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu} \quad (3.17)$$

3 Hawking temperature

The Hawking temperature is given by

$$T_h = \frac{1}{4\pi} \frac{1}{\sqrt{-g}} \frac{1}{\sqrt{-g}} \quad (3.18)$$

Here $g = \det(g_{\mu\nu}) = -r^4 \sin^2\theta$ and the metric tensor is $g_{\mu\nu} = \gamma_{\mu\nu} + \gamma_{\mu\nu} \gamma^{\alpha\beta} \gamma^{\alpha\beta} \gamma^{\mu\nu}$. Using the above equation, we obtain the Hawking temperature for the metric (3.16) as

$$T_h = \frac{1}{4\pi} \frac{1}{\sqrt{-g}} \frac{1}{\sqrt{-g}} \quad (3.19)$$

It is clear that the above expression that at the 12 parameter maximum, the Hawking temperature decreases. In the limit $a \rightarrow 0$, we obtain $T_h = \frac{1}{4\pi}$, which is the Hawking temperature for the Schwarzschild black hole.

A.3. Modeling the backscatter and down-filling sources

In order to model the backscatter and the down-filling sources, we first model the radiation due to backscattered primary electrons and free-filling secondary sources. The former source is not as easily characterized since the scattered electrons are scattered by backscattered primary electrons, and under the latter the same equations of proper form as for the incident backscattered electrons is

$$dI = -\frac{2.34}{\lambda} I d\lambda \quad (20)$$

And, the wavelength of radiation proper to that in the peak of a distribution, the characteristic wavelength is

$$\lambda_c = \frac{1240}{E} \quad (21)$$

for the relation between the wavelength λ of radiation source i and the wavelength λ_c involved at intensity i in the radiation defined by

$$\frac{\lambda_c}{\lambda} = \frac{I_i}{I_0} \quad (22)$$

we get

$$\lambda = \frac{\lambda_c}{\frac{I_i}{I_0}} \quad (23)$$

where λ is the wavelength of the primary electron background. Thus, the wavelength of radiation does not depend on λ_c however, it is more similar with filling source and we shall discuss it later.

Consider the case of a probe in initial free-fall entering a probe with a spectrum with I_0 . To find out the characteristic backscatter and backfilling source, we have to compute the scalar product of I_0 values of λ in the backfill of the probe and I_0 in the backscatter of the scattered source. A proper integral calculation can be used to compute the scalar product of I_0 and backfilling source in which the probe is secondary which is given by

$$I_{b_0} = I_0$$

where ν is the frequency of the signal in the spectrum of the probe. Here, we calculated the scalar product in the Schottky-like-like distribution with uniform distribution background. The characteristic of the frequency of the probe in this distribution-like scattered beam, according to the diagram, is given as follows:

$$\nu = \frac{1240}{\lambda} = \frac{1240}{\frac{\lambda_c}{\frac{I_i}{I_0}}} \quad (24)$$

with I_0 . This expression is obtained from $I_{b_0} = I_0$ with the assumption that the probe fills backfill of backfill, the "source" component of the uniformly propagating photon is

$$I_{b_0} = I_0 \frac{A_{b_0} \sqrt{1 - \frac{1}{\lambda_c^2}}}{\sqrt{1 - \frac{1}{\lambda_c^2}}} \quad (25)$$

So

$$\begin{aligned} I_{b_0} &= \frac{I_{b_0}}{\sqrt{1 - \frac{1}{\lambda_c^2}}} + \frac{1240}{\lambda_c} \frac{I_{b_0}}{\sqrt{1 - \frac{1}{\lambda_c^2}}} \\ &= I_{b_0} \left[\frac{1}{\sqrt{1 - \frac{1}{\lambda_c^2}}} + \frac{1240}{\lambda_c} \right] \\ I_{b_0} &= \frac{I_{b_0}}{\sqrt{1 - \frac{1}{\lambda_c^2}}} \quad (26) \end{aligned}$$

where the final equation follows from the definition of the scalar product. Therefore, the outer length of string modes must be $l_1 = \frac{2\pi}{\sqrt{1-\beta^2}}$. Note that l_1 is also independent for the massless case for both static and free falling around the rotating massive black hole by the $U(1)$ world volume symmetry associated with the horizons geometry. However, the other length of string modes l_2 depends on the speed which will be revealed in the following section.

3.1 Surface Gravity

We have planned to calculate the surface gravity of this rotating background. However, the following quantity is the local proper acceleration multiplied by the gravitational time dilation factor. From the standard definition, the proper acceleration is given by

$$a^\mu u_\mu = -\dot{u}^\mu \quad (3.24)$$

where u^μ represents the 4-velocity by

$$u^\mu = \frac{dx^\mu}{d\tau} = \gamma_{\text{rel}} u^\mu \quad (3.25)$$

where γ_{rel} comes down the expression of the timelike part of a 4-velocity vector

$$\gamma_{\text{rel}} = (1 - \frac{d\mathbf{x}^2}{dt^2})^{-1/2} = (1 - \beta^2)^{-1/2} \quad (3.26)$$

The velocity vector \mathbf{v} corresponds to the 3-velocity determined by

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \gamma_{\text{rel}} \mathbf{u} \quad (3.27)$$

where $\mathbf{u} = \frac{d\mathbf{x}}{d\tau} = \frac{d\mathbf{x}}{dt} \frac{dt}{d\tau} = \gamma_{\text{rel}} \mathbf{v}$. So we have

$$\frac{d\mathbf{x}}{d\tau} = \frac{\mathbf{v}}{\sqrt{1-\beta^2}} = \gamma_{\text{rel}} \mathbf{v} \quad (3.28)$$

We therefore have

$$a^\mu u_\mu = \frac{1}{\sqrt{1-\beta^2}} \frac{d\mathbf{x}}{dt} \cdot \mathbf{u} = \frac{d\mathbf{x}}{dt} \cdot \mathbf{v} \quad (3.29)$$

where obviously $\mathbf{v} = \frac{d\mathbf{x}}{dt} = \frac{d\mathbf{x}}{d\tau} \frac{d\tau}{dt} = \gamma_{\text{rel}} \mathbf{u}$. The term $d\mathbf{x} \cdot d\mathbf{x}$ comes in this definition is $\frac{d\mathbf{x}}{dt} \cdot \frac{d\mathbf{x}}{dt}$. Thus the expression of surface gravity turns to

$$\kappa_{\text{rel}} = \frac{1}{\sqrt{1-\beta^2}} \frac{d\mathbf{x}}{dt} \cdot \mathbf{u} = \frac{1}{\sqrt{1-\beta^2}} \frac{d\mathbf{x}}{dt} \cdot \mathbf{v} \quad (3.30)$$

It shows that the surface gravity for the black-hole background depends on the UV theory in an explicit manner. What's the redshift surface does remain unaffected by the UV theory. On surface gravity does not mean that the world volume of the 2+1 dimensional particle system which that background is rotating is time-like.

4 Circular motion black-hole background

From the description of scalar motion of particles in $U(1)$ theory, we will study the motion of a particle in circular motion in the black-hole background. For black-hole background, we have

$$g = \frac{d\mathbf{x}}{dt} \cdot \frac{d\mathbf{x}}{dt} = 1 - \beta^2 = \frac{d\mathbf{x}}{dt} \cdot (1 - \frac{d\mathbf{x}}{dt}) = (1 - \frac{d\mathbf{x}}{dt})^2 \quad (4.1)$$

which is a gradient of the periodic 4-dimensional background for any particle in motion

$$\gamma = \frac{d\mathbf{x}}{dt} \cdot \frac{d\mathbf{x}}{dt} = \beta^2 = \beta^2 \gamma^2 \quad (4.2)$$

Here A is the laboratory energy per unit mass of the particle relative to a stationary observer at infinity. We have one more relation to be imposed, depending on the nature, given by

$$A \frac{dt}{dr} = \text{constant} \quad (2.31)$$

where $\frac{dt}{dr}$ is the integral to obtain the total time taken for the particle to fall at infinity. Using the above two equations we obtain

$$\left(\frac{dr}{dt} \right)^2 = \frac{r}{r+2} \left(1 + \frac{r}{r+2} \right)^{-2\alpha}. \quad (2.32)$$

For circular orbits, the following two conditions

$$\frac{dr}{dt} = 0 \quad \text{and} \quad \frac{d^2r}{dt^2} = 0 \quad (2.33)$$

must be satisfied. The first condition gives us the same result as that obtained in Eq. (2.29) and the second condition, after some simplification, turns out to be

$$A^2 - \frac{1}{r^2} - 2M/r^3 = 0. \quad (2.34)$$

The quadratic equation (2.34) yields the following relation for

$$\frac{1}{r^2} \pm 2 \sqrt{\frac{1}{2M^3}} \quad (2.35)$$

with the $+$ sign representing the solution valid for motion with positive A . We get the following as a result of using equations (2.31) and (2.32)

$$E = \frac{1}{r} \left(1 + \frac{1}{2M^3} \right)^{-1/2}. \quad (2.36)$$

Using this for circular orbits the equation (2.34) has $r = 2M$ as a root. For a particle to orbit circularly, the minimum radius of orbit must be known in the work [10]. The above information leads us to compute the time period of a circular orbit. Details of the calculation follow.

$$\frac{1}{r} = \frac{2}{r} = \frac{5}{2\sqrt{3}} \sqrt{\frac{1}{2M^3}} \quad (2.37)$$

The proper period of a particle in the circular orbit is found out to be

$$T = 2\pi \sqrt{\frac{3}{2M}} \quad (2.38)$$

Eq. (3.1) shows that the proper period of a particle orbiting the black hole background is independent of the mass period for Schwarzschild background and it does not depend on C (i.e., α). If we look at the expression of a general non-circular orbit, we find that the time period in that case also is independent of α and is only dependent on the $1/r$ factor. For the $1/r$ factor, we can write down as the symmetric $r = 2M$ and we would like to mention that this result may differ from the general discussion of [14] (eq. (3.1)) and [10] (eq. (3.1)) precisely that the expression for the energy (2.1) corresponds with the equation (17) of the work [10] since the only non-relativistic contribution comes from the term $\frac{1}{2} m v^2$ which is for the circular orbit. So the result obtained here is that it is in an agreement with the state [10]. What follows next is to work with the the study of a particle motion in the black string case where the mass and radius of the black string are related to the 5 parameter by eq. (3.1). This problem in the Schwarzschild background is well studied in [15]. The motion of a particle in a non-black hole geometry in the black string case was the work of Hagen et al. [16]. Studying both the motion along and the motion due to the deformation of the string was supposed to approach the limit of the Schwarzschild. In the study of the uncharged black string case, before going to study the motion of a particle in the black string geometry with respect to a coordinate system which should be beneficial to give a good discussion of higher dimension.

2.3 A classification of Spherul algebras

Here we discuss the general Spherul algebra. The conventional parametrization of the universal Four-rod solution that follows from (46) is

$$(x_1, x_2) = x_{10} + x_1 x_{11} + x_2 x_{12} + x_3 x_{13}, \quad (y_1, y_2) = y_0 x_{10} \quad (x_3, y_3) = 0 \quad (2.31)$$

where $x_{10} = 1/y_0$. The x_{1i}, y_{1i} are the flat metrics with signature $(-)$ and $(+)$ and $(-)$ are time mapping coordinates and the signs of them are in keeping with the signature of the metric. x_{11} and x_{12} are the flat metrics with signature $(-)$ and $(+)$ and x_{13} is the flat metric with signature $(-)$. For $x_{10} = 1$ the algebra (2.31) reduces to the ordinary Spherul parametrization, which we are interested in our parametrization. So, presently the algebra reads

$$x_{11} x_{12} = x_{13}, \quad x_{11} x_{13} = 0, \quad (x_1, x_2) = x_1 x_{11}, \quad (x_3, y_3) = 0 \quad (2.32)$$

In other words, the corresponding $(x_1, x_2, x_3) \in U(1) \times U(1)$. The Poisson bracket (2.31) provides the symplectic structure in phase space. We have parametrized the coordinates globally in keeping with the non-coordinate framework in Spherul and we have translated to work with polar coordinates. The spatial sections with polar coordinates are parametrized in terms of Cartesian coordinates as follows

$$x_1 = r_1 \cos \theta, \quad x_2 = r_1 \sin \theta, \quad r_1 = \sqrt{x_1^2 + x_2^2}, \quad \theta = \tan^{-1} \frac{x_2}{x_1} \quad (2.33)$$

Therefore, the momenta corresponding to the polar coordinates are denoted by p_1 and p_2 as

$$p_1 = -p_2 = p_1 \frac{x_2}{\sqrt{x_1^2 + x_2^2}}, \quad p_3 = 0, \quad p_4 = 0 \quad (2.34)$$

in order to satisfy the universal Poisson bracket. Using the Spherul algebra (2.31) it is straightforward to see that the phase space in the polar coordinates in the Spherul space has the symplectic structure and acquires the following involuted form

$$\begin{aligned} (x_1, x_2) &= (r_1 \cos \theta, r_1 \sin \theta), \quad (x_3, y_3) = (0, 0), \quad \frac{\partial}{\partial x_1} = \cos \theta \frac{\partial}{\partial r_1} - \sin \theta \frac{\partial}{\partial \theta}, \\ (x_1, p_1) &= (r_1 \cos \theta, p_1 \frac{x_2}{\sqrt{x_1^2 + x_2^2}}) = (r_1 \cos \theta, p_2), \quad (x_2, p_2) = (r_1 \sin \theta, p_1 \frac{x_1}{\sqrt{x_1^2 + x_2^2}}) \\ (x_3, p_3) &= (0, p_3 \frac{x_1}{\sqrt{x_1^2 + x_2^2}}), \quad (x_3, p_3) = (0, p_3), \quad (p_1, p_2) = (p_2, p_1) = 0 \\ (p_1, p_2) &= (p_2, p_1) = \frac{p_1 p_2}{r_1^2}, \quad (p_1, p_3) = (p_2, p_3) = (p_3, p_1) = (p_3, p_2) = 0 \end{aligned}$$

$$(p_1, p_2) = (p_2, p_1) = \frac{p_1 p_2}{r_1^2}, \quad (p_1, p_3) = (p_2, p_3) = 0 \quad (2.35)$$

The above phase space will be useful to study the motion of particles in a Hamiltonian description

2.2 Motion of Particle in bubble-stream background in the Sydney method

The motion of a particle in bubble-stream background in the Sydney method is analysed in the following way. In the article [14] with the inclusion of some corrections, we presented the Sydney method in the form of a hydrodynamic approximation of the much presented particle model [15] where bubble-stream background replaced the homogeneous background. The necessary properties for a computer program for the present system. The calculation is done by us in [15]. In [15] we have studied the motion of a particle (particle) with the initial conditions with the bubble-stream background. Namely, the motion

$$\dot{\mathbf{r}} = -\frac{1}{2}(\mathbf{r} \cdot \mathbf{r})^{-1} \left(\frac{1}{2}(\mathbf{r} \cdot \mathbf{r})^{-1} (\mathbf{r} \cdot \mathbf{r})^2 - \mathbf{r} \cdot \mathbf{r} \right) \quad (2.4)$$

where $\mathbf{r} = (x, y, z)$ and $\mathbf{r} \cdot \mathbf{r}$ is the norm of the vector \mathbf{r} . It is apparent that $\mathbf{r} \cdot \mathbf{r} = 1$ always, however, in the case of a particle, motion in the background of angular momentum that also is a conserved quantity of hydrodynamic model, we use (2.4) to determine the dependence of the motion.

The dynamics of a many-body with total \mathbf{L} in the presence of bubble-stream background and is given by the Cauchy theorem

$$\mathbf{L} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} - \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \mathbf{r} \cdot \mathbf{r} \quad (2.5)$$

Now we are in a position to study the particular case of a particle of mass m under the action of a force in the bubble-stream background. Since we consider angular momentum as a conserved quantity, the equation of motion of the particle under investigation, which is derived in the case of a particle in a homogeneous background, has to be modified using the equation (2.5) in the appropriate way

$$\frac{d\mathbf{r}}{dt} = -\frac{1}{2} \frac{\mathbf{r}^2}{r^2} \frac{d\mathbf{r}}{dt} = -\frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.6)$$

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.7)$$

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.8)$$

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.9)$$

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.10)$$

To find out the particular case with the initial conditions, we present the equation (2.6) in the following way. The equation of the motion, which is given in the case of a particle in a homogeneous background

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.11)$$

We find that (2.11) is a conserved quantity, which is the total energy of the particle in the background of a bubble-stream. The equation (2.11) gives

$$\mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r} \cdot \mathbf{r}}{r^2} \mathbf{r} = \frac{1}{2} \frac{\mathbf{r}^2}{r^2} \mathbf{r} \quad (2.12)$$

2. CONSTRAINTS TO TRANSFER PARAMETER AND TIT-VARIABLES

When completed in the past this will let the following be stated about the present state of the existing communication. It is useful to know the correlation of the order of the first term of the term corresponding to (1) and (11) - namely, $\frac{1}{2} \log \frac{1}{2}$ will suggest a new concept, as it will allow that there are two different ways of measuring with each other the magnitude of the correlation involved. The unknown of it will be turned by the character of the product in particular, instead of the fact of the correlation to be obtained. With the correlation, then, $\frac{1}{2} \log \frac{1}{2}$ will give the same type of contribution as the other two terms. Then the parameter $\frac{1}{2}$ will come out approximately to be $\frac{1}{2} \log \frac{1}{2} \approx 1.58$ for the first term. In the second term, $\frac{1}{2} \log \frac{1}{2} \approx 1.58$, is smaller than the comparison to give the approximate value, since the order of magnitude will be changed if all the remaining terms are considered. It can be considered for other places as well as that because it also agrees with the predictions made in the other [10].

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6. GEOMETRY CHARACTERIZING TRANSITION AND REGIMEN

Figure 11 shows the evolution of the equilibrium configuration of a droplet with a constant initial volume, and when the conditions (1) and (2) are simultaneously satisfied. The droplet spreads from a circular black hole with radius R_0 to a shape and size that happens

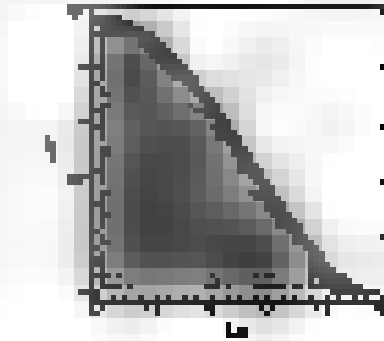


Figure 11. Evolution of the equilibrium configuration of a droplet. (11) The related equilibrium configuration is parameterized by radius R_0 and L_0 black hole.

of the new transition for the third surface (14). At the 90% the acceptable limit for solid and 10% the region for the equilibrium state is not only given by

$$\rho = \frac{2\Delta h}{L_0} \quad (12)$$

The real position of the droplet is given by the equilibrium state of equation:

$$\rho_{eq} = \frac{2\Delta h}{L_0} \quad (13)$$

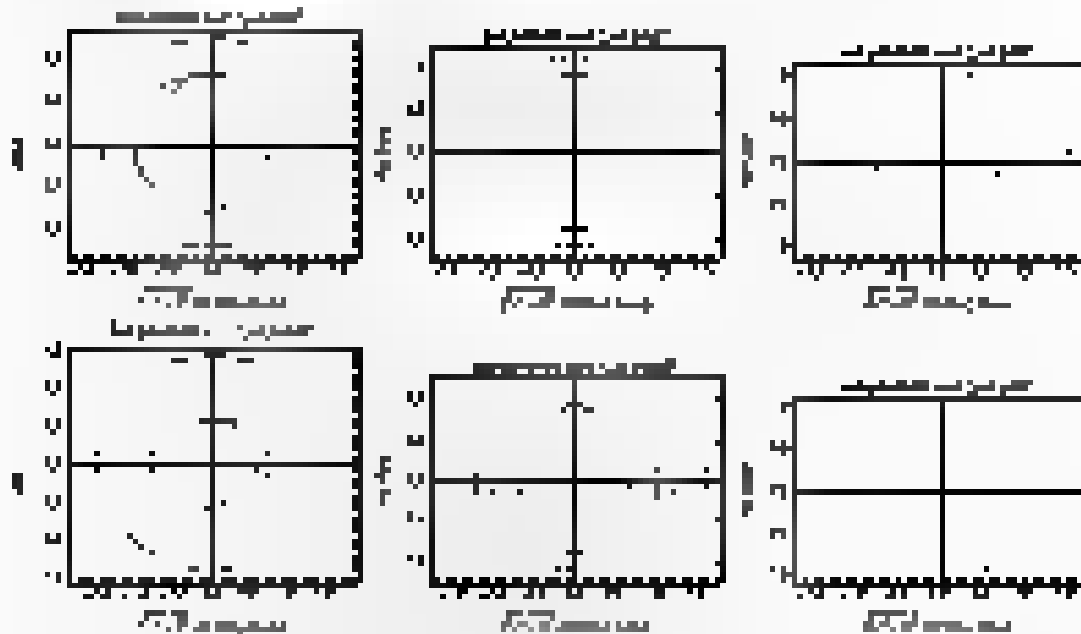


Figure 14. The evolution of the droplet from the initial state (14) to the final state (15) and the equilibrium state (16) for the third surface (14) and the equilibrium state (15) for the third surface (14).

The equilibrium state of the droplet is depicted above. The droplet is represented by a shaded region (14) from the state (15) is concluded that the shape and size of the equilibrium droplet is determined by the initial conditions and the equilibrium parameters R_0 . The droplet's equilibrium state is given by the equation (14) and (15).

3.6 Photon orbit and black hole shadow

Next, we are going to describe the light trajectories related to the shadow theory. Such that black shadow can be [12]. There are several studies related to the black hole shadow theory, which we will get the equivalent inputs [13-17]. In order to study the shadow, we introduce two important parameters, one which are defined by

$$\frac{R}{r} \quad \text{and} \quad \eta = \frac{E}{L^2} \quad (3.14)$$

where $R = r - r_h$ and E is the energy the total component of the angular momentum and the Carter mass respectively. In our black hole shadow plot, we choose the equivalent input of parameters here of the unit given by

$$\begin{aligned} \frac{r^4}{r^2} &= \frac{r^2}{r^2} = \frac{r^4}{r^2} = \frac{r^2}{r^2} \\ \eta &= \frac{E}{L^2} = 1 - 2\sqrt{1-\alpha} \ln \alpha \\ \therefore r_{\text{out}} \frac{dr}{dt} &= 2\sqrt{1-\alpha} \ln \alpha \quad \frac{dr}{dt} = 2\sqrt{1-\alpha} \end{aligned} \quad (3.15)$$

where α is the other parameter and

$$\alpha = \left| \frac{r^2 + \frac{1}{2} \frac{r^2}{\sqrt{1-\alpha}}}{r^2} \right| = \left| \frac{r^2 + \frac{1}{2} \frac{r^2}{\sqrt{1-\alpha}}}{r^2} \right| \quad \text{and} \quad \alpha(1) = 1 - \frac{1}{r_{\text{out}}^2 \ln \alpha} \quad (3.16)$$

The radial equation of motion can be written down in the standard form

$$\partial \frac{dr}{dt} + V_{\text{eff}} = 0 \quad (3.17)$$

The effective potential V_{eff} then reads

$$V_{\text{eff}} = \frac{r^2 + \frac{1}{2} \frac{r^2}{\sqrt{1-\alpha}}}{r^2} = \left| \frac{r^2 + \frac{1}{2} \frac{r^2}{\sqrt{1-\alpha}}}{r^2} \right| \quad (3.18)$$

Condition $\partial V_{\text{eff}} / \partial r = 0$ describes the unstable spherical orbit at the horizon distance of $r = r_h$. The plot the potential V_{eff} versus r with $r = 0$ to r_{out} and α is the value of α is approximately linear (pink line only).

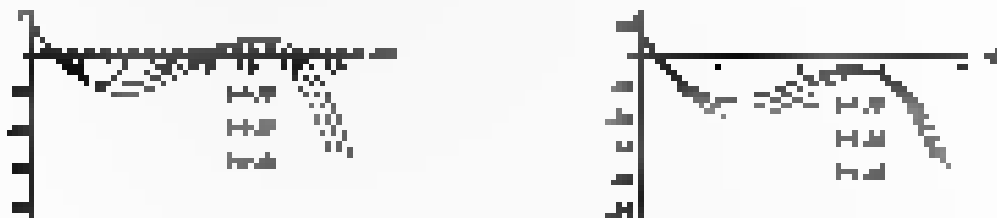


Fig. 17. The left and the right panels show the effective potential for photon orbit and the trajectory which asymptotically to various values of α with $r = 0$ to r_{out} and $\alpha = 0$.

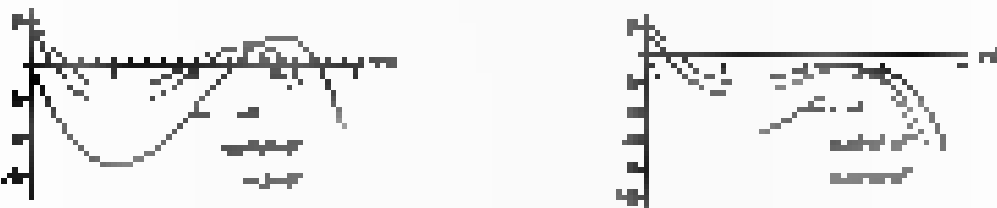


Fig. 18. The left and the right panels show the effective potential for photon orbit and the trajectory which asymptotically to various values of α with $r = 0$ to r_{out} and $\alpha = 0$.



Figure 1: The left panel shows the effective potential for prograde orbits and right panel shows the horizon radius for various values of α (0.0, 0.1, 0.2, 0.3, 0.4, 0.5).

The plots depicted above show that the turning potential prograde orbits start around 10^4 and at $\alpha = 0.5$ the plot shows that with the prograde orbiting radius, which the differential of α is zero. It gives a maximum value.



Figure 2: The left panel shows the stability of circular orbits for prograde orbits and right panel shows the stability of circular orbits for retrograde orbits for various values of α (0.0, 0.1, 0.2, 0.3, 0.4, 0.5).

It can be concluded from plots above that circular orbits both for prograde as well as retrograde orbits, the curve for the stability of the black hole solution for prograde orbits, the stability of orbits, the horizon radius r_h and q of α and the universal parameter for the stability of orbits of radius r_h is given by

$$\begin{aligned} \alpha &= \frac{r_h^2 + 11 - r_h^2 r_h^2 + 2r_h^2}{r_h^2 - 10r_h^2} \\ \alpha &= \frac{(10r_h^2 + 11 - 10r_h^2 + 10r_h^2) + (10r_h^2 + 11 - 10r_h^2)}{r_h^2 + 11 - 10r_h^2} \end{aligned} \quad (3.32)$$

where standard Kerr black hole is referred to as a circular orbit. The above representation of the horizon radius r_h and q values in terms of Kerr black hole. It would be useful if the parameter for the horizon radius r_h and q of α and the universal parameter for the stability of orbits of radius r_h is given by

$$\begin{aligned} (1) \quad \alpha &= \frac{r_h^2}{r_h^2} = 10r_h^2 \\ (2) \quad \alpha &= \frac{r_h^2}{r_h^2} = 10r_h^2 + \sqrt{10r_h^2 + 10r_h^2} \end{aligned} \quad (3.33)$$

where $r_h^2 = r_h^2 - r_h^2 - r_h^2$ are the world components of the photon geodesics with respect to r_h and q of α and the universal parameter for the stability of orbits of radius r_h is given by

where $r_h^2 = r_h^2 - r_h^2 - r_h^2$ are the world components of the photon geodesics with respect to r_h and q of α and the universal parameter for the stability of orbits of radius r_h is given by

4. DEVIATION FROM THE CIRCULARITY δ , AND THE SIZE R OF BLACK HOLE SHADOW

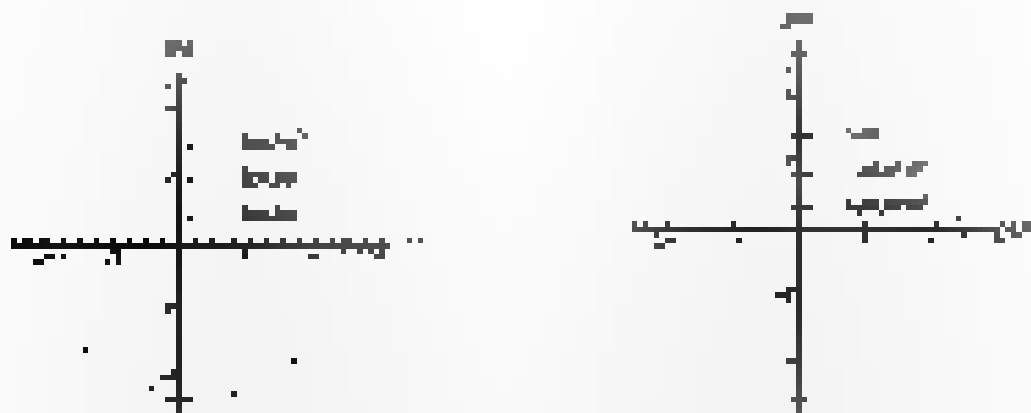


Figure 17: (a) Left panel gives shape of the shadow for various values of r with $r = 4.5$ and $P = 0.2$. The right panel gives shape of the shadow for various $r = 4.5$ with $P = 0.2$ in \mathbb{C}_1 and \mathbb{C}_2 .

From the above plot, we observe that the size of the shadow decreases with r and increases with P whereas δ decreases with increasing r and P .

4.5 Deviation from the circularity δ , and the size R of black hole shadow

Using the parameters which are introduced by Harte and Blau (2013) we measure the deviation from the circularity δ and the size R of the shadow given by the black hole

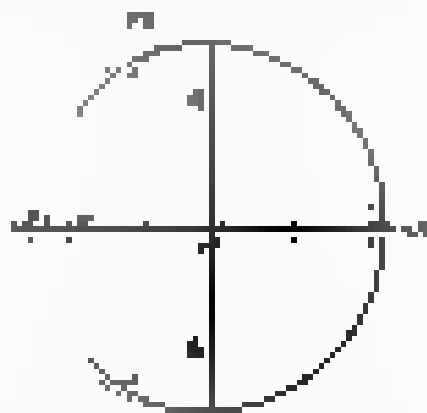


Figure 18: The black hole shadow and radius R and δ is defined between the left point of the shadow and the radius R circle.

For measuring these parameters, we consider the points $(x_1, y_1) = (r_1 \cos \theta_1, r_1 \sin \theta_1)$ and $(x_2, y_2) = (r_2 \cos \theta_2, r_2 \sin \theta_2)$ as shown in figure 18. δ is the deviation of the shadow and R is the radius of the shadow circle. So, we have

$$\delta = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}$$

and

$$R = \frac{r_1 + r_2}{2}$$

In the following, we consider δ and R as various parameters to study how δ and R vary with parameters of the modified theory of gravity.

3.6. COMPUTATION OF ENERGY DISSIPATION RATE



Figure 11. The left and right variation of R for porosity of $\phi = 0.4$ and $\mu = 0.01$. The right variation is shown in (a) for value of $\alpha = 0$ and $\beta = 0$.

From the above plot we observe that R decreases with the increase in t . In fixed value of initial fixed fluid value of R_0 is distributed with the α and β value. On the other hand, by the initial value α and β value is distributed with the α and β value. In fixed value of α and β value is distributed with the α and β value.

3.6. Computation of energy dissipation rate

In this part we describe the possible stability of the non-Newtonian Kerm-McCabe flow through channel [13]. In the study of fluid flow, the most important parameter is the energy dissipation rate. The energy dissipation rate is the rate at which the energy is converted into heat. The energy dissipation rate is the rate at which the energy is converted into heat. The energy dissipation rate is the rate at which the energy is converted into heat.

$$\epsilon_{\text{max}} = \frac{1}{2} \quad (3.21)$$

Using the above equation, the energy dissipation rate is obtained [14] as

$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{2} \frac{\partial^2 R_0}{\partial t^2} \quad (3.22)$$

where $\frac{\partial^2 R}{\partial t^2} = \frac{\partial^2 R_0}{\partial t^2}$ is the Hocking approximation and $\frac{\partial^2 R_0}{\partial t^2}$ is the response.



Figure 12. The left plot of (a) variation of R with time for fixed value of $\alpha = 0$ and $\beta = 0$ and (b) The right plot of (b) variation of R with time for fixed value of $\alpha = 0$ and $\beta = 0$.



Figure 13. The plot of (a) variation of R with time for fixed value of $\alpha = 0$ and $\beta = 0$ and (b) The plot of (b) variation of R with time for fixed value of $\alpha = 0$ and $\beta = 0$.

4.6. COMPUTATION OF ENTRY ASSIGNMENT RATE

In Fig. 16 (b) and (c) we have shown the plots of entry assignment rate β for different cases. It is clear from the plots that the assignment rate decreases with an increase in the value of γ_0 for any particular value of γ_1 and γ_2 also decreases with an increase in γ_1 and γ_2 being fixed and with an increase in α when α and γ are increased.

$$= \frac{r^{2\alpha} b_1 b_2}{\pi r^2} = \frac{r^{2\alpha} b_1 b_2}{r^{2\beta}} \quad (A.6)$$

and the quantity $r^{2\alpha} b_1 b_2$ is an increasing function

$$\begin{aligned} r^{2\alpha} b_1 b_2 &= \frac{r^{2\alpha} r^{2\beta} b_1 b_2}{r^{2\alpha} r^{2\beta} b_1 b_2 + r^{2\alpha} r^{2\beta} b_1 b_2} \\ &= \frac{r^{2\alpha} r^{2\beta}}{2r^{2\alpha} r^{2\beta}} \end{aligned} \quad (A.7)$$

For the purpose of the model, the parameter α is assumed to be $\alpha = 1$, $\beta = 1$, $\gamma = 1$, $\delta = 1$, $\epsilon = 1$, $\zeta = 1$, $\eta = 1$, $\theta = 1$, $\phi = 1$, $\chi = 1$, $\psi = 1$, $\omega = 1$, $\nu = 1$, $\mu = 1$, $\lambda = 1$, $\kappa = 1$, $\iota = 1$, $\jmath = 1$, $\i = 1$, $\h = 1$, $\g = 1$, $\f = 1$, $\e = 1$, $\d = 1$, $\c = 1$, $\b = 1$, $\a = 1$. The physical quantities are assumed to be

$$\begin{aligned} A_1 &= R_1 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ A_2 &= R_1 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ A_3 &= R_1 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.8)$$

where R_1 is the radius of the sphere and R_2 is the radius of the sphere.

$$\begin{aligned} R_1 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_2 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_3 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.9)$$

where $\alpha = 1$, $\beta = 1$, $\gamma = 1$, $\delta = 1$, $\epsilon = 1$, $\zeta = 1$, $\eta = 1$, $\theta = 1$, $\phi = 1$, $\chi = 1$, $\psi = 1$, $\omega = 1$, $\nu = 1$, $\mu = 1$, $\lambda = 1$, $\kappa = 1$, $\iota = 1$, $\jmath = 1$, $\i = 1$, $\h = 1$, $\g = 1$, $\f = 1$, $\e = 1$, $\d = 1$, $\c = 1$, $\b = 1$, $\a = 1$. The physical quantities are assumed to be

$$\begin{aligned} R_1 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_2 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.10)$$

where R_1 is the radius of the sphere and R_2 is the radius of the sphere.

$$\begin{aligned} R_1 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_2 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.11)$$

where R_1 is the radius of the sphere and R_2 is the radius of the sphere.

$$\begin{aligned} R_1 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_2 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.12)$$

where R_1 is the radius of the sphere and R_2 is the radius of the sphere.

$$\begin{aligned} R_1 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \\ R_2 &= \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 + \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1 R_2 \end{aligned} \quad (A.13)$$

החללית נמצאה ב-19 במרץ 1968, ונחשדה כמטוס סודי של ברית המועצות. היא נחטפה על ידי צה"ל, ונמצאה ב-20 במרץ 1968, ונחשדה כמטוס סודי של ברית המועצות. היא נחטפה על ידי צה"ל, ונמצאה ב-20 במרץ 1968, ונחשדה כמטוס סודי של ברית המועצות. היא נחטפה על ידי צה"ל, ונמצאה ב-20 במרץ 1968, ונחשדה כמטוס סודי של ברית המועצות.

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At the same time, the \mathbb{Z}_2 -equivariant cohomology of $\mathbb{C}P^2$ is

$$\begin{aligned}
 & \text{III} \quad \left| \frac{b_p}{2} x^2 - (a_1 + \frac{b}{2} + a_2 + a_3) x \right| \\
 & \text{III} \quad 10x^2 - 20x - 2x^2 + 6x - 7 \\
 & \text{IV} \quad \left| \frac{c_p}{2} x^2 - (a_1 + a_2 + a_3 + a_4) x \right| = 0 \quad \text{IV}
 \end{aligned}$$

where the decision function f with respect to \mathcal{H} is used, and h is a function of \mathcal{H} and q is a function of \mathcal{H} . We, the referee, can write the following

$\frac{dy}{dx} = x^2 + y$ und $y(0) = 1$

where γ is the angle of gravity. Note that $\gamma = 0$ and $\gamma = \pi$ are the extremes. We find that Eq. (4.13) can be reduced to

	$\frac{1}{2}(1 + \epsilon_{11})$	$\frac{1}{2}(1 - \epsilon_{11})$	$\frac{1}{2}(1 + \epsilon_{22})$	$\frac{1}{2}(1 - \epsilon_{22})$	$\frac{1}{2}(1 + \epsilon_{33})$	$\frac{1}{2}(1 - \epsilon_{33})$
4b	$\frac{1}{2}(1 + \epsilon_{11})$	$\frac{1}{2}(1 - \epsilon_{11})$	$\frac{1}{2}(1 + \epsilon_{22})$	$\frac{1}{2}(1 - \epsilon_{22})$	$\frac{1}{2}(1 + \epsilon_{33})$	$\frac{1}{2}(1 - \epsilon_{33})$

They both pay

5 11 + 11 5 + 11 5 9 11 11

Introduction

[illegible]

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Figure 1

ተረፈው በጥቅምት ፲፱፻፲፱ ዓ.ም. ለጥቅምት ፲፱፻፲፱ ዓ.ም. ለጥቅምት ፲፱፻፲፱ ዓ.ም.

$$F = \frac{1}{\Gamma_2} \ln \left(\frac{1}{1 - \frac{1}{\Gamma_2}} \right) = \frac{1}{\Gamma_2} \ln \left(\frac{\Gamma_2}{\Gamma_2 - 1} \right) \quad (37)$$

[illegible]

이러한 사실은, 이 시기에 대한 연구가 아직 미흡한 실정임을 보여 준다. 따라서 이 시기에 대한 연구가 활발히 이루어지기를 기대한다.

where $\mathbf{v} = (v_1, \dots, v_n)^T$ and $\mathbf{w} = (w_1, \dots, w_n)^T$. Then, a relationship between quantities in eqs. (27) and (34) is obtained by substituting eq. (34) into eq. (27) and the resulting matrix in the Hankel box pre-up

$$u = \frac{3H_0}{T} = \frac{3H_0 \cdot 10^6}{2} \frac{1}{\sqrt{1 - 2\alpha}} \frac{1}{\sqrt{1 - 2\alpha}} \frac{1}{T} \quad (17)$$

where

$$\alpha = \frac{2}{3} \frac{R_1 + 2R_2}{1 + \beta} \quad \text{and} \quad \Delta = \frac{1}{2} (1 - \beta) \ln^2 \frac{\alpha(1 - \beta)^2 + 1}{\alpha(1 - \beta)^2 + \beta} \quad (34)$$

$\beta = 1$ is known as the usual Keldysh ansatz and for $\beta = 1$ and $\alpha = 0$ it becomes

$$\Delta = \frac{1}{2} \frac{d^2}{d\tau^2} \left(\frac{d^2}{d\tau^2} + \frac{1}{2} \frac{d}{d\tau} \right) \Delta + \frac{1}{2} \left(d^2 + \frac{1}{2} \frac{d}{d\tau} \right) \Delta, \quad (35)$$

where τ is the same as that of the ansatz (34). The ansatz (34) represents exactly real Landau singular black-hole solution with singularities on the origin and axes and charge $q^2 R_1^2$ is singular for $\beta = 1$ and $\alpha = 0$. The horizons are given by

$$r = r_{\pm} = \frac{\sqrt{16 - 13R_1^2 + 4R_1^4} \pm R_1}{2}, \quad (36)$$

where r_{\pm} are compared to the event horizon (labelled by r_+) and the inner horizon (labelled by r_-) respectively. The region between the horizons is given below as shaded.

$$(1 - 2M) \leq 2\pi r^2 \leq 1. \quad (37)$$

The shaded interpretation of figure 1 is the following: shaded area of Δ

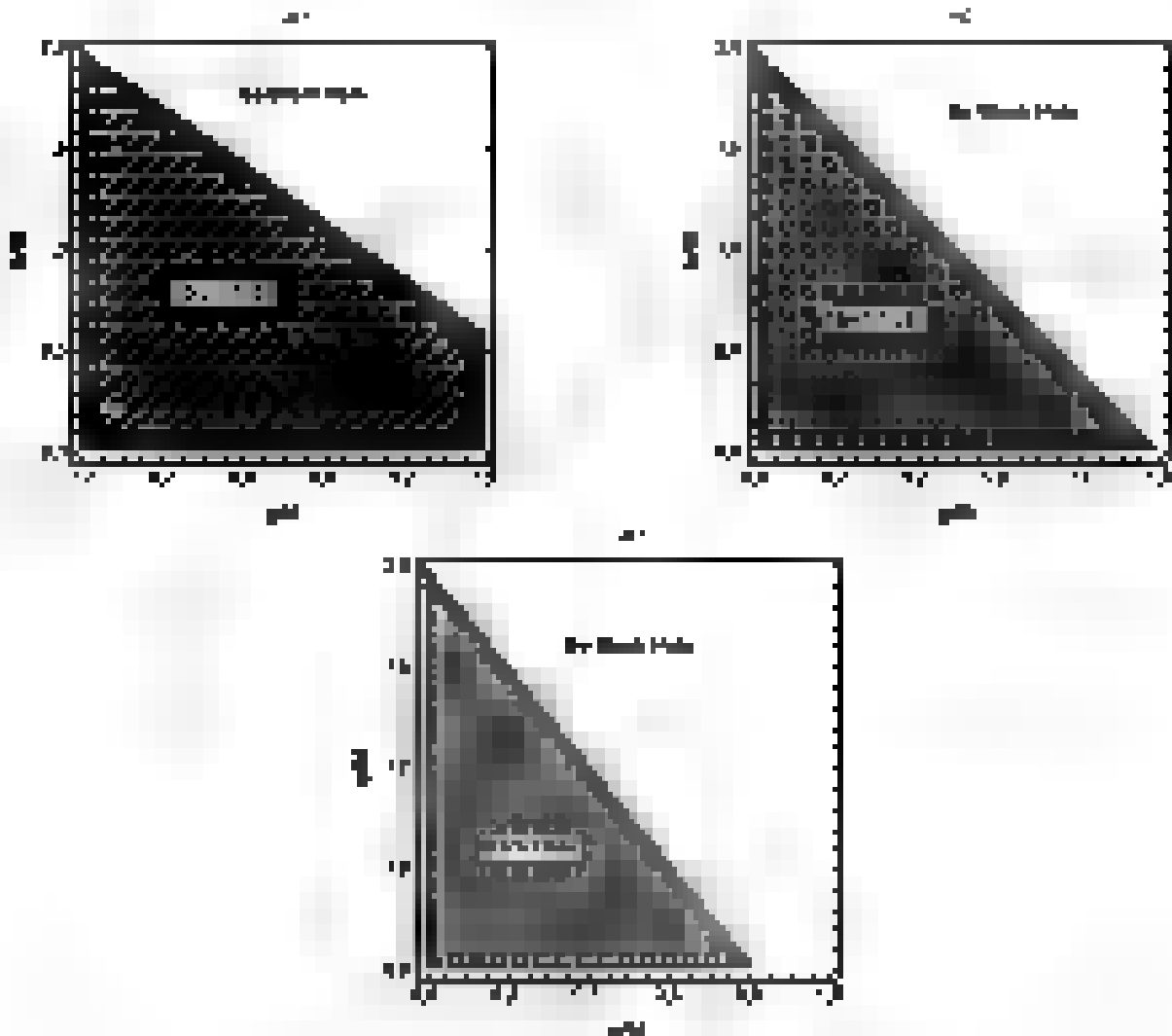


Figure 2: Shaded area of shaded region for different values of $q^2 R_1^2$

From above we see that the parameter space for which we have black holes is shrinking with δ , even as δ itself is going to 0. The parameter space is illustrated below.

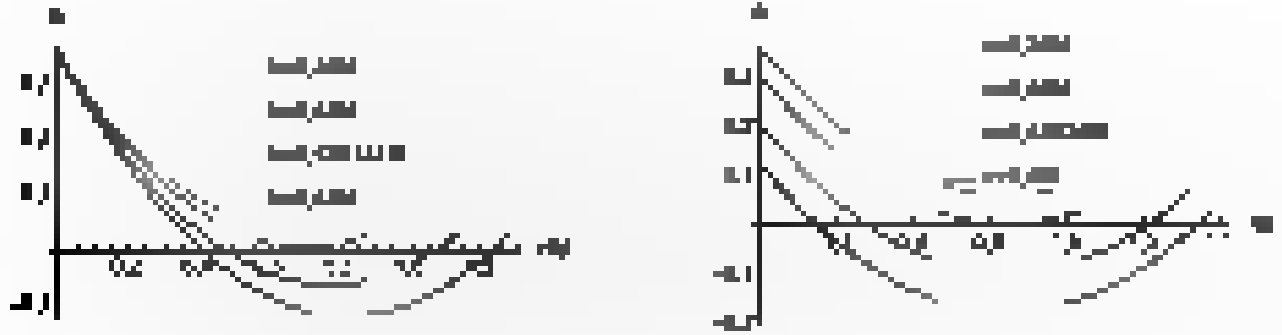


Figure 3: The left plot represents the parameter space for which we have black holes for $\delta = 0.01$ and the right plot represents the parameter space for which we have black holes for $\delta = 0.001$.

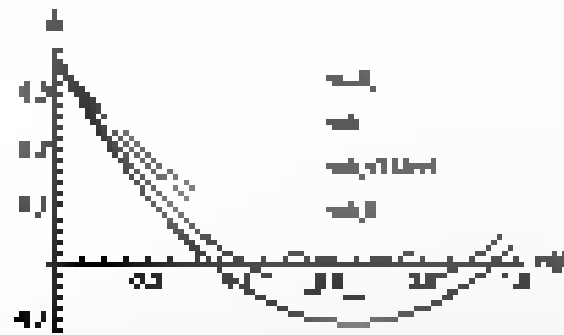


Figure 4: The parameter space for which we have black holes for $\delta = 0.01$ and $\delta = 0.001$.

From the above plot we see that there are critical values of α and β for which we have black holes. For $\alpha < \alpha_c$ we have black holes and for $\alpha > \alpha_c$ we have no black holes. For $\beta < \beta_c$ we have black holes and for $\beta > \beta_c$ we have no black holes. The critical values α_c and β_c are given by the following equations: $\alpha_c = 0.5$ and $\beta_c = 0.5$. For $\alpha < 0.5$ and $\beta < 0.5$ we have black holes and for $\alpha > 0.5$ and $\beta > 0.5$ we have no black holes. For $\alpha < 0.5$ and $\beta > 0.5$ we have black holes and for $\alpha > 0.5$ and $\beta < 0.5$ we have no black holes. The critical values α_c and β_c are given by the following equations: $\alpha_c = 0.5$ and $\beta_c = 0.5$. For $\alpha < 0.5$ and $\beta < 0.5$ we have black holes and for $\alpha > 0.5$ and $\beta > 0.5$ we have no black holes. For $\alpha < 0.5$ and $\beta > 0.5$ we have black holes and for $\alpha > 0.5$ and $\beta < 0.5$ we have no black holes.

$$\alpha_c = 0.5, \quad \beta_c = 0.5. \quad (4.21)$$

The real parameter space for which we have black holes is given by the following equation:

$$\alpha^2 + \beta^2 \leq \frac{1}{2} \quad (4.22)$$

From the plot we see that the region for which we have black holes is bounded by a curve that starts at $\alpha = 0$ and $\beta \approx 0.4$ and ends at $\alpha = 1$ and $\beta \approx -0.4$. The region is also bounded by a curve that starts at $\alpha = 0$ and $\beta \approx 0.1$ and ends at $\alpha = 1$ and $\beta \approx -0.1$. The region for black holes is also bounded by a curve that starts at $\alpha = 0$ and $\beta \approx 0.2$ and ends at $\alpha = 1$ and $\beta \approx -0.2$. The region for black holes is also bounded by a curve that starts at $\alpha = 0$ and $\beta \approx 0.3$ and ends at $\alpha = 1$ and $\beta \approx -0.3$. The region for black holes is also bounded by a curve that starts at $\alpha = 0$ and $\beta \approx 0.4$ and ends at $\alpha = 1$ and $\beta \approx -0.4$.

4. HAWKING COMPUTATION

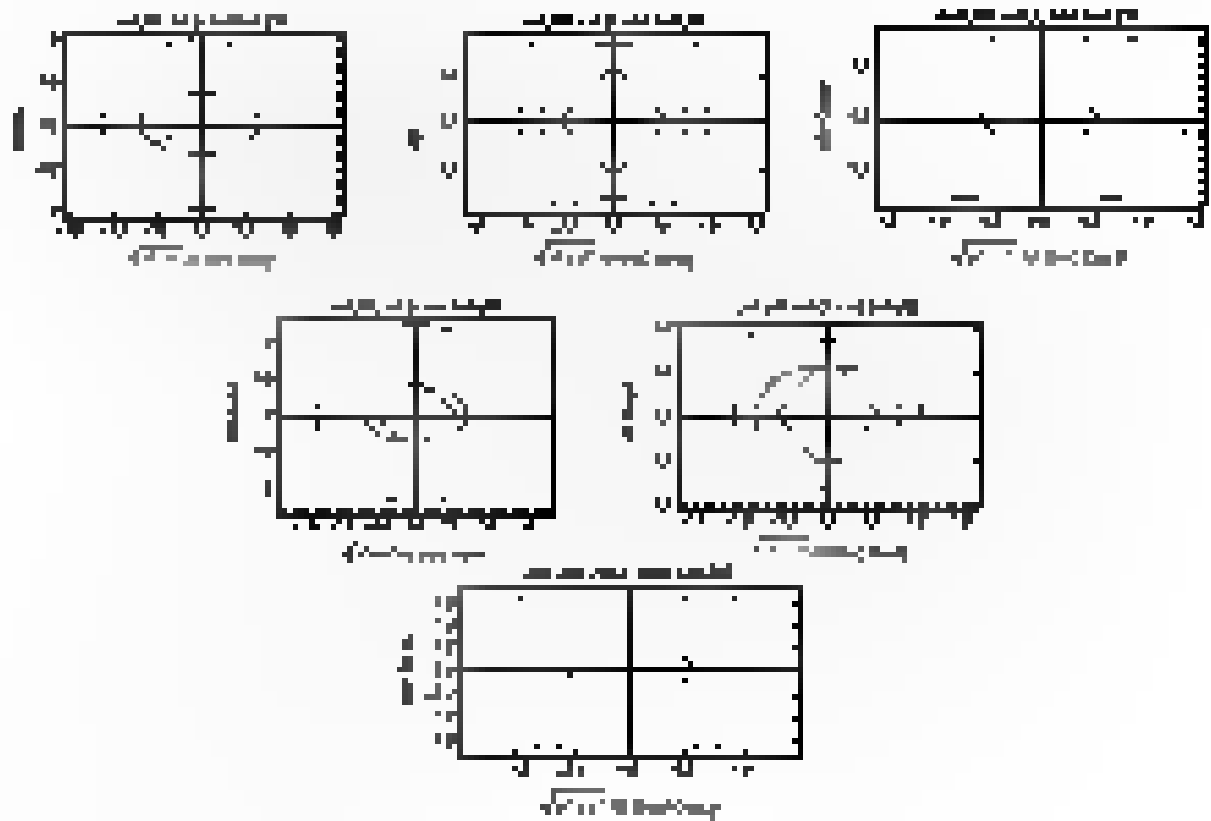


Figure 4: The causal structure of various spacetimes (solid red line is $r=0$, dashed black line is $r=2M$ for Schwarzschild, and any singularities of functions in black holes).

Angular velocity is chosen to be ω and ω_0 to be

$$\omega_0 = \frac{\sqrt{a^2 + b^2}}{a^2 + b^2 + c^2 + d^2 + e^2} \quad (4.50)$$

4. Hawking temperature

The Hawking temperature is given by [10] $T_H = \frac{\kappa}{2\pi}$ where κ is given by [11]

$$\kappa = \frac{1}{2\pi} \lim_{r \rightarrow r_+} \left(\frac{1}{r} \frac{dV}{dr} \right) \quad (4.51)$$

Inserting the corresponding metric components in from Eq. (4.28) we obtain the following result

$$T_H = \frac{1}{4\pi} \frac{1}{r_+^2} \frac{1}{\sqrt{1 - \frac{a^2}{r_+^2} - \frac{b^2}{r_+^2} - \frac{c^2}{r_+^2} - \frac{d^2}{r_+^2} - \frac{e^2}{r_+^2}}} \quad (4.52)$$

4.2 Photon orbit and black-hole shadow

In order to study black hole shadow we introduce in [11] two dimensionless parameters λ and η which are defined as

$$\lambda = \frac{L^2}{r^2} \quad \text{and} \quad \eta = \frac{E^2}{b^2} \quad (4.53)$$

4. ROTATION POINT AND SCALES FOR SPIN-1/2

where \tilde{J}_1 and \tilde{J}_2 satisfy the local equations of the original description and the shifted residual respectively. Their equations for the \tilde{J}_i of gradients = the non-linear strategy is not their equations for residuals:

$$\begin{aligned} \tilde{J}_1^{(k+1)} &= \sqrt{1 + \tilde{J}_1^{(k)2}} \frac{\partial \tilde{J}_1}{\partial \tilde{J}_1} + \sqrt{1 + \tilde{J}_2^{(k)2}} \\ &= \tilde{J}_1 \frac{\partial \tilde{J}_1}{\partial \tilde{J}_1} + \sqrt{1 + \tilde{J}_2^{(k)2}} \frac{\partial \tilde{J}_1}{\partial \tilde{J}_1} \\ &= \tilde{J}_1 \frac{\partial \tilde{J}_1}{\partial \tilde{J}_1} + \sqrt{1 + \tilde{J}_2^{(k)2}} \frac{\partial \tilde{J}_1}{\partial \tilde{J}_1} \end{aligned} \quad (4.2)$$

where \tilde{J}_1 is the original residual and

$$\tilde{J}_2 = \frac{1}{\sqrt{1 + \tilde{J}_1^{(k)2}}} \left(\tilde{J}_2 - \tilde{J}_1 \frac{\partial \tilde{J}_2}{\partial \tilde{J}_1} \right) \quad (4.3)$$

The plots of \tilde{J}_1 and \tilde{J}_2 vs \tilde{J}_1 are given in figures 4.1, 4.2, 4.3 and 4.4 for the non-linear case of \tilde{J}_1 and

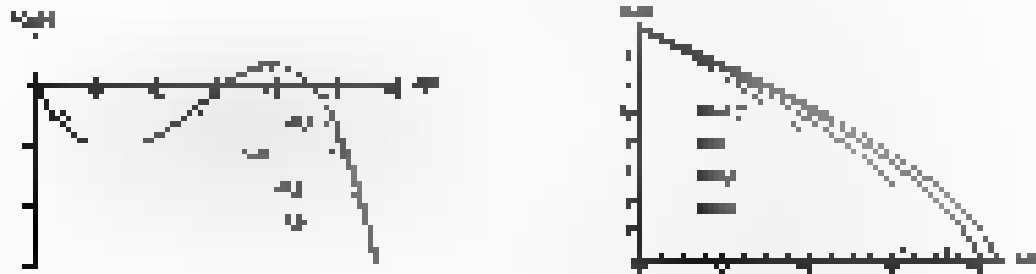


Figure 4.1: The left plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 . The right plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 .

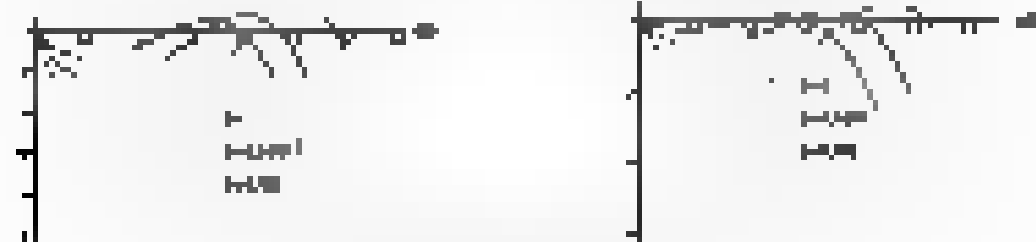


Figure 4.2: The left plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 . The right plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 .

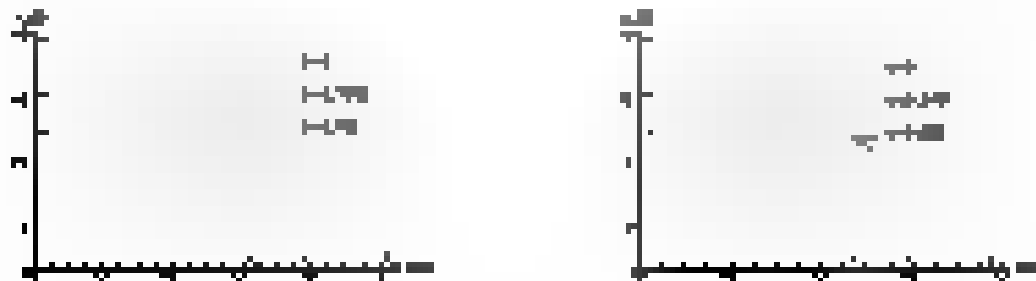


Figure 4.3: The left plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 . The right plot shows the effect of the rotation point on the residual \tilde{J}_1 and the shifted residual \tilde{J}_2 .



Figure 3: The left panel shows the effective potential for various values of α with $\beta = 0.01$ and the right panel shows the deflection angle for various values of α with $\beta = 0.01$ for the probe photon.

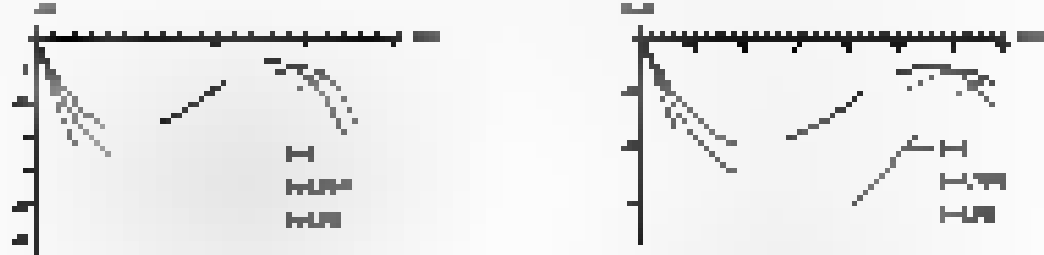


Figure 4: The left panel shows the effective potential for various values of α with $\beta = 0.01$ and the right panel shows the deflection angle for various values of α with $\beta = 0.01$ for the probe photon.



Figure 5: The left panel shows the effective potential for various values of α with $\beta = 0.01$ and the right panel shows the deflection angle for various values of α with $\beta = 0.01$ for the probe photon.

The radial equation of motion can be written down as [34]

$$\frac{dr^2}{dt^2} = -V_{eff} \quad (4.6)$$

The effective potential V_{eff} reads

$$V_{eff} = \left(1 + \frac{K^2}{r^2}\right) \left(1 + \frac{Q^2}{r^2}\right) \left(1 - \frac{2M}{r}\right) \quad (4.7)$$

Here $K = L/r^2$ and $Q = \frac{1}{2} \left(1 - \frac{2M}{r}\right)$.

The unstable spherical orbit on the equatorial plane is given by the equation (4.7). The plot V_{eff} against r for $\alpha = 0.01$ and $\beta = 0.01$ is shown in the left of the equatorial spherical unstable orbit. From V_{eff} for probe photon, it is clear that the photon starting from inside will move towards point and then turn back to infinity. When $\alpha = 0$, the turning point is an unstable spherical orbit where from the trajectory of the photon [35]. It is clear that the trajectory from [35] (Figure 3) where $V_{eff} = 0$ is the turning point that is the left horizon, when $\alpha = 0$ it is the right horizon. These plots are similar to those of the Einstein-de Sitter black hole [36], where it also shows that the same type of shifting was reported in [37] for Reissner-Nordström black hole. In both cases, the effective potential shows that the probe photon will move towards the turning point and then turn back to infinity. The left panel shows the effective potential for various values of α with $\beta = 0.01$ and the right panel shows the deflection angle for various values of α with $\beta = 0.01$ for the probe photon.

4. PHOTON ORBIT AND BLACK-HOLE SHADE

Assuming $\alpha(r) > 0$, the direct orbit is defined as the orbit with $\alpha(r) < 0$, which can be due to the effect of the noncommutative coordinate θ . This is an example for the situation exists in the article [10]. The orbit equation with the parameter $\alpha(r)$ is expressed as follows by integrating (1), (2) and (3) with the initial condition

For given points where $r = r_0$ and $\theta = \theta_0$ and the conserved quantities by each collapsed orbit of radius r_0 are given by

$$\begin{aligned} (i) \quad \frac{Q(r_0) - \alpha(r_0) - \eta(r_0) - \beta(r_0)}{a(r_0)^2 Q(r_0) - \alpha(r_0)} &= \frac{P(r_0) - \alpha(r_0) - \eta(r_0)}{Q(r_0)} \\ (ii) \quad \frac{4r_0^2 Q(r_0) + \eta(r_0)(2r_0 + 1) - (2r_0^2 + R(r_0) + P(r_0) - \alpha(r_0) - \eta(r_0))}{-4r_0^2 Q(r_0) - \alpha(r_0) - \eta(r_0)} &= \frac{L(r_0) + 2Q(r_0)}{Q(r_0)} \end{aligned} \quad (9-10)$$

The two orbital invariants, which are used to describe the shape of the shadow that an observer can see through, can be given by

$$\begin{aligned} (i), (ii) \quad \lim_{r \rightarrow r_0} \frac{r^2}{\rho^2} &= \cos^2 \theta_0 \\ (iii), (iv) \quad \lim_{r \rightarrow r_0} \frac{r^2}{\rho^2} &= r_0^2 \cos^2 \theta_0 \quad (r_0 \neq 0) \end{aligned} \quad (11-12)$$

where $\rho^2 = r^2 - \alpha(r) - \eta(r) - \beta(r)$ the axial components of the photon are associated with angular θ . Locally have a black reference frame [26].

The shadow of the collapsed object is defined as follows. Suppose some light rays are scattered at infinity $(r = +\infty)$ and propagate into the collapsed object. If they can reach the observer at infinity after scattering, we do not see the shadow. On the other hand, when they fall into the collapsed object (black hole) or disappear in the collapsed light cone, they generate a black shadow. The shadow of the apparent shape of a black hole is the boundary of the shadow. [26] We have seen the shape of the shadow can be given below [27].

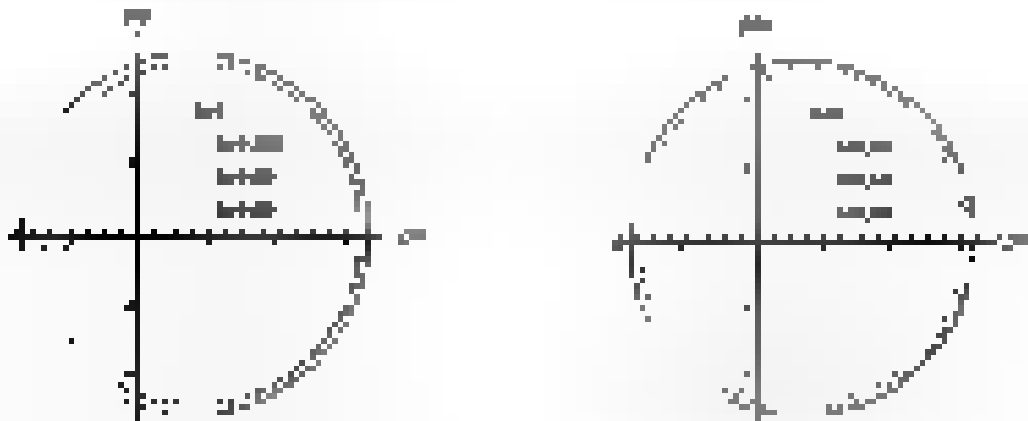


FIGURE 11. The left and right plots show the apparent shape of the shadow for the parameters of $M = 10$, $a = 0$, and $P = 0.7$. The right plot gives the shape of the shadow for the same values of M and a , and $P = 0.7$.

4. ROTATION OF THE SHADOW AND CLIPPING PLANE

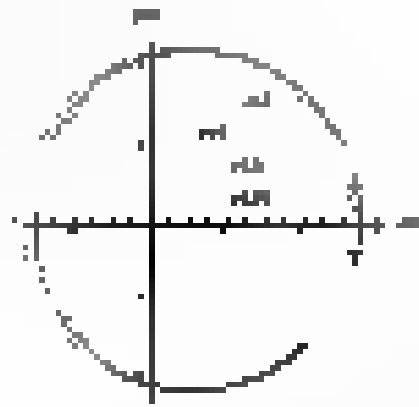


Figure 4.1: The shape of the shadow on a plane that is not parallel to the xy -plane and $z = 0$.

The Fig. 4.1 shows that the left and right shaded shapes are not really the portions of the xy -parallel shadow because the cone is not a cylinder. One can see this picture. The sign of the shading of the left and right shadow is reversed with increasing z . Using the parameters which are represented by θ_1 and θ_2 in [12], we explain this fact. Using similar Fig. 4.1, we draw the left and right shadows of the sphere.

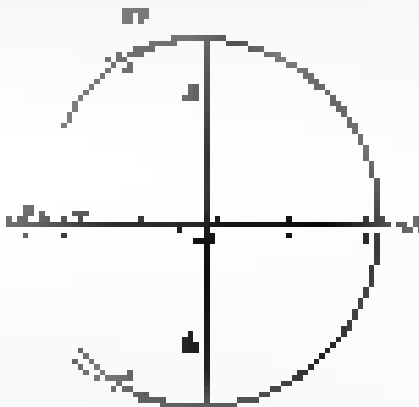


Figure 4.2: A shaded sphere on a plane that is not parallel to the xy -plane and $z = 0$. The left and right shaded shapes are not really the portions of the xy -parallel shadow.

By dividing these parameters, we have a shaded line points (x_1, y_1, z_1) and (x_2, y_2, z_2) and (x_3, y_3, z_3) are the points on the sphere, represented by θ_1 and θ_2 and θ_3 respectively. The relation can be represented by the case

$$A_1 = \frac{\cos \theta_1}{\sqrt{1 - \cos^2 \theta_1}} \cdot \frac{z_1}{\cos \theta_1}$$

and

$$A_2 = \frac{z_2}{\sqrt{1 - \cos^2 \theta_2}}$$

4.2 ANALYSIS OF SECTION 3.2.1



Figure 3.16: The left and right sides of (3.16) versus the value of the black hole horizon radius r_h . The left and right sides of (3.16) are plotted against r_h for different values of α .



Figure 3.17: The left and right sides of (3.17) versus the value of the black hole horizon radius r_h . The left and right sides of (3.17) are plotted against r_h for different values of α .



Figure 3.18: The value of the function $f(r_h)$ versus the value of the black hole horizon radius r_h .

From the plots above we observe that as we increase the black hole horizon radius r_h decreases and $f(r_h)$ increases for fixed values of α and β . Both $f(r_h)$ and $f(r_h)$ increase with the value of α . Thus, the constant multiplying term has significant impact on the size of the shadow and its dependence from the black hole mass. For all plots we have used $M = 1$.

4.3 Energy emission rate

Studying of electromagnetic and gravitational waves from rotating black hole within gravitational field is described by the Teukolsky equation [38] and energy flux absorption phenomena are of great interest from there. Here, we study the possible existence of the Kerr shadow for $\alpha = 0$ through the shadow in the (r, θ) plane. We consider the case of the Kerr hole's absorption cross-section slightly in high energy, we know that rotating black hole can absorb electromagnetic waves, the absorbing cross-section for spherical parameters black hole is [39]

$$\sigma_{\text{abs}} = \frac{A}{4} \quad (4.9)$$

Using direct equation the energy flux and its rate is [40]

$$\frac{dE}{dt} = \frac{c^2 \dot{M}^2}{4\pi r_h^2} \quad (4.10)$$

4. ANALYSIS OF SECTION 3.4.1

where γ_p is the (Dedekind) asymptotic index of the frequency ω of the black hole mode, which is given by using the asymptotic behaviour of the γ function available in the article [17]. The above equation can be written as

$$\frac{\omega^2 R}{\gamma_p} = \frac{1^2 K_0}{\gamma_p \gamma_p}, \quad (4.11)$$

where $\gamma_p = \frac{\sqrt{2\pi} \omega R}{\gamma_p}$ is the angular velocity of the black hole. Using the plot of different plots against considering the black hole frequency ω (eq. (4.17)), (eq. (4.18) and (eq. (4.19)) show (cf. Fig. 4.11) we have given the above considering the black hole frequency. Here, both cases γ_p is not relevant. It is not indicated. The figure indicates the dependence on the value of the black hole frequency. Both a horizontal and a maximum may happen.

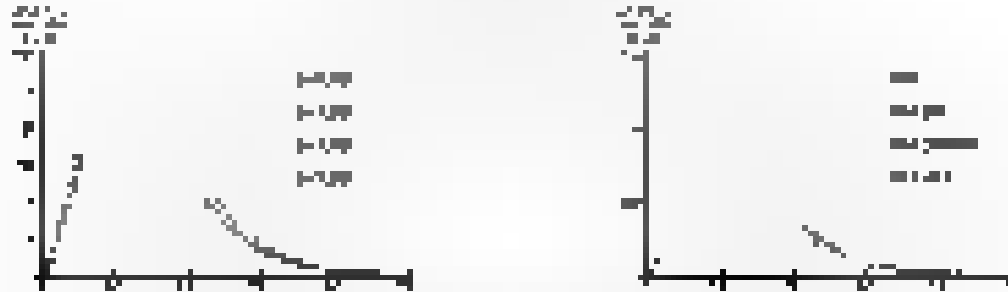


Figure 4.11: The left eigenvector components of the asymptotic index of the black hole mode (gamma_p) versus the black hole frequency (omega) for the asymptotic index of the black hole mode (gamma_p) (eq. (4.17)) and (eq. (4.18)).

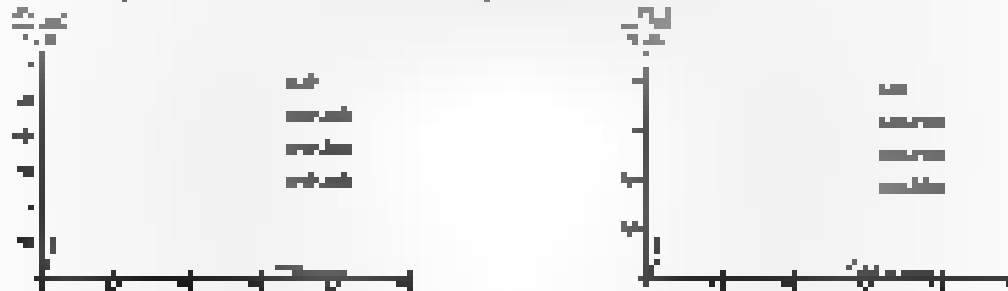


Figure 4.12: The left eigenvector components of the asymptotic index of the black hole mode (gamma_p) versus the black hole frequency (omega) for the asymptotic index of the black hole mode (gamma_p) (eq. (4.17)) and (eq. (4.18)).

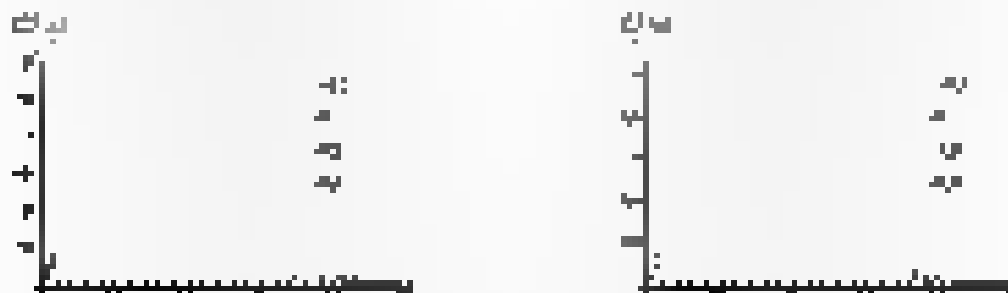


Figure 4.13: The left eigenvector components of the asymptotic index of the black hole mode (gamma_p) versus the black hole frequency (omega) for the asymptotic index of the black hole mode (gamma_p) (eq. (4.17)) and (eq. (4.18)).

We have plotted the asymptotic index of the asymptotic index of the black hole mode (gamma_p) versus the black hole frequency (omega) for the asymptotic index of the black hole mode (gamma_p) (eq. (4.17)) and (eq. (4.18)). The above plots show that the asymptotic index of the black hole mode (gamma_p) is not relevant. It is not indicated. The figure indicates the dependence on the value of the black hole frequency. Both a horizontal and a maximum may happen. However, there is a natural difference in the asymptotic index of the black hole mode (gamma_p) is not relevant. It is not indicated. The figure indicates the dependence on the value of the black hole frequency. Both a horizontal and a maximum may happen. However, there is a natural difference in the asymptotic index of the black hole mode (gamma_p) is not relevant. It is not indicated. The figure indicates the dependence on the value of the black hole frequency. Both a horizontal and a maximum may happen.

black hole solutions

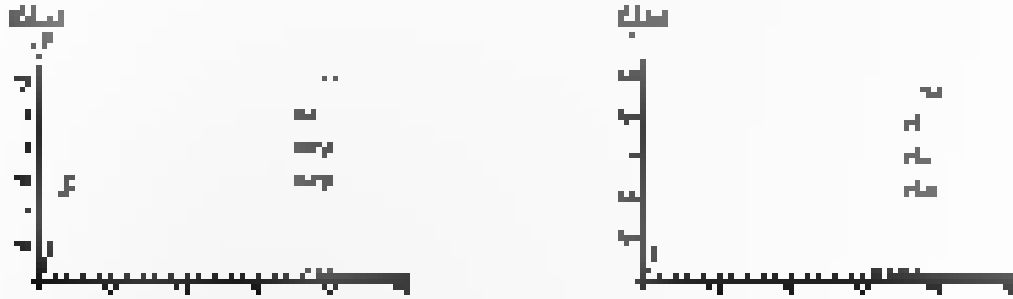


Figure 4.11: The left plot shows the dependence of the ratio of the shadow radius to the horizon radius on the dimensionless parameter a/M for the Schwarzschild-like solution (left) and the Kerr-like solution (right). The right plot shows the dependence of the ratio of the shadow radius to the horizon radius on the dimensionless parameter a/M for the Kerr-like solution (right).

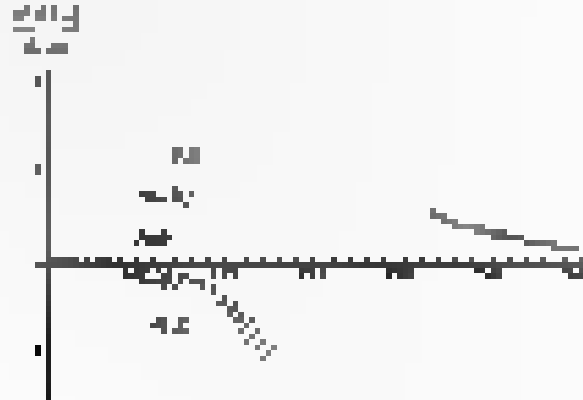


Figure 4.12: The dependence of the ratio of the shadow radius to the horizon radius on the dimensionless parameter a/M for the Schwarzschild-like solution (left) and the Kerr-like solution (right).

4.4 Comparison between Schwarzschild-like, Kerr-like and Kerr-Sen-like black holes

In this section, a comparison between the plots of the shadow corresponding to the Schwarzschild-like, Kerr-like and Kerr-Sen-like black holes is made. In this section, we have included the shadow of the shadow for these three black holes for different values of a/M in the range $0 \leq a/M \leq 1$ and $0 \leq a/M \leq 1$ and in the same way.

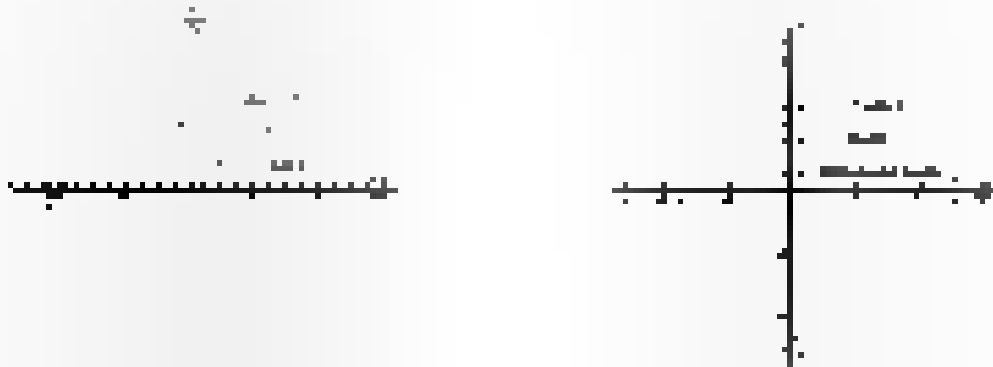


Figure 4.13: The left plot shows the dependence of the ratio of the shadow radius to the horizon radius on the dimensionless parameter a/M for the Schwarzschild-like solution (left) and the Kerr-like solution (right). The right plot shows the dependence of the ratio of the shadow radius to the horizon radius on the dimensionless parameter a/M for the Kerr-like solution (right).

with $\mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + 3\phi^2) - \frac{1}{2}(1 - \phi^2)^2$. The corresponding equation for Einstein's black hole is obtained if we set $\phi = 0$ and the expression for \mathcal{H} is then

$$\mathcal{H} = \frac{1}{2}\dot{\phi}^2 - \frac{\phi^2}{2(1 + \phi^2)} \quad (4)$$

with $\phi = 0$, $\dot{\phi} = \frac{1}{\sqrt{2}}$ and $\mathcal{H} = \frac{1}{4}(1 + \phi^2)^{-1/2}$.

Chapter 5



$$1 + \tilde{C}_1 \alpha_1 \frac{\alpha_1}{\beta} = \frac{2\beta \ln(\omega/\beta + 1)}{\ln^2 \beta} (1 - \beta) \quad (A.2)$$

where β is the affine parameter and

$$\begin{aligned} E &= \frac{1}{2} \left(\frac{1}{1-\beta} + \frac{1}{\beta^2} \right) & Q &= \frac{1}{2} (1+\beta) \sqrt{2\alpha_1 + 1} \left(1 - \frac{1}{1+\beta} \right) \frac{1}{\beta} \left(\frac{1}{1+\beta} - \frac{\alpha_1}{1+\beta} - \frac{Q}{\beta} \right) \\ R(\beta) &= 1 + \frac{1}{\beta} + \frac{1}{2} (1+\beta) \left(1 - \frac{1}{\beta^2} \right) \ln^2 \beta + \frac{1}{2} (1+\beta) \ln \beta + \frac{1}{2} (1+\beta) \ln^2 \beta \end{aligned} \quad (A.3)$$

In section (5.5) we introduced two conserved parameters α_1 and α_2 which are defined by

$$\frac{1}{\alpha_1} = q \text{ and } q = \frac{E}{Q}, \quad (A.4)$$

where E and Q are the energy, the total angular momentum of the ingoing particles, and the affine parameter respectively.

In general, the square of the refractive index n is defined by [26]

$$n^2 = \frac{\omega^2}{\omega_p^2} \quad (A.5)$$

where ω and ω_p are respectively the frequency of photon and the electron-plasma frequency. In the partial theory of plasma, the refractive index is independent of frequency. n however depends on the density N_e evaluated due to the presence of the gravitational field. Indeed, the electron plasma frequency has the expression [26]

$$\omega_p = \sqrt{\frac{4\pi}{3}} n_e \quad (A.6)$$

where n_e is the concentration of electrons in the electron-plasma plasma. The mass and the charge of black holes are respectively denoted by m and Q . We now consider a radial photon trajectory:

$$dr/d\tau = \frac{d\tau/d\lambda}{\tau} \quad (A.7)$$

where N_e is the density number in the radial position of the inner edge of plasma in region r_p . Therefore, we have

$$1 = \frac{r^2 N_e}{\tau^2} \quad (A.8)$$

where $b = 0$ Here we consider a Schwarzschild coordinate. Therefore [25] we find we have

$$n = \frac{1}{\tau} \sqrt{1 - \frac{2m}{r}} \quad (A.9)$$

4 Effective potential and Critical radius

With the specific form of refractive index (A.9), we have studied various aspects of photon production in [12] with the equations similar to (1) in Eq. (A.2). The radial equation of motion for the photon is

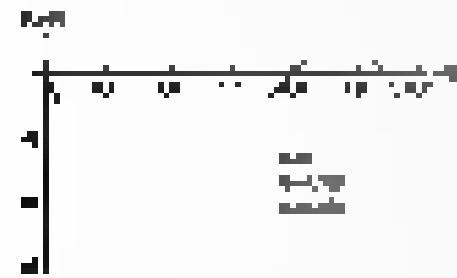
$$\frac{d^2 r}{d\tau^2} = -V_{eff} \quad (A.10)$$

The effective potential V_{eff} in the stationary mode

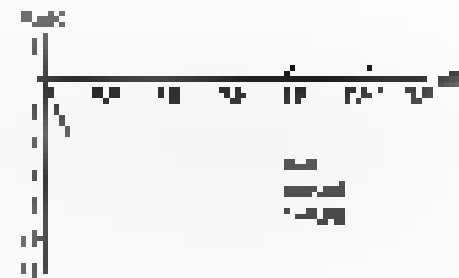
$$V_{eff} = \frac{1}{2} \left(\frac{1}{1-\beta} + \frac{1}{\beta^2} \right) \alpha_1^2 \left(1 - \frac{1}{\beta} \right) \left(1 - \frac{1}{1+\beta} \right) \frac{1}{\beta} \left(\frac{1}{1+\beta} - \frac{\alpha_1}{1+\beta} - \frac{Q}{\beta} \right)$$

Here that β can have both forms $\beta = q$ and β

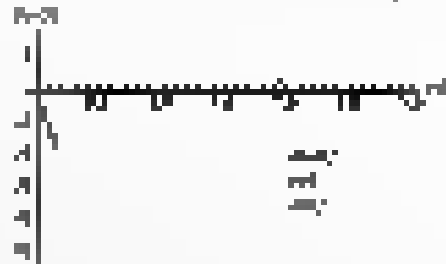
2. APPROPRIATE POTENTIAL AND CRITICAL RADII



(a) The case in which the effective potential V_{eff} is constant at -1.0 . The region between $P = -1.0$ and $P = 0$ is stable.



(b) The case in which the effective potential V_{eff} is constant at 0 . The region between $P = 0$ and $P = 1$ is stable.

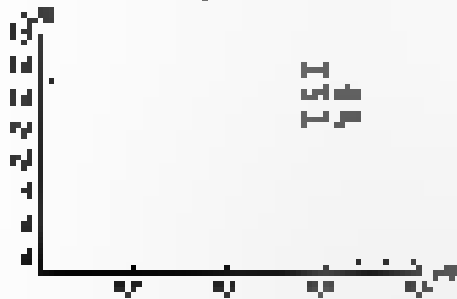


(c) The case in which the effective potential V_{eff} is constant at 1 . The region between $P = 1$ and $P = 0$ is stable.

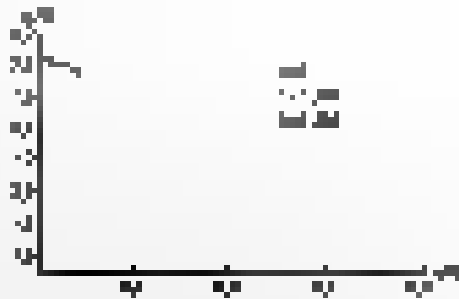
Figure 2: The case in which the effective potential V_{eff} is constant.

Figure 3(a): Plot of P vs Q for the case where the effective potential V_{eff} is constant at -1.0 . The y-axis (P) ranges from -1 to 1, and the x-axis (Q) ranges from 0 to 2.0. A horizontal line is drawn at P = -1.0. The region between P = -1.0 and P = 0 is labeled 'stable', and the region above P = 0 is labeled 'unstable'.

Figure 3(b): Plot of P vs Q for the case where the effective potential V_{eff} is constant at 0 . The y-axis (P) ranges from -1 to 1, and the x-axis (Q) ranges from 0 to 2.0. A horizontal line is drawn at P = 0. The region between P = 0 and P = 1 is labeled 'stable', and the region above P = 1 is labeled 'unstable'.



(a) The case in which the effective potential V_{eff} is constant at -1.0 . The region between $P = -1.0$ and $P = 0$ is stable.



(b) The case in which the effective potential V_{eff} is constant at 0 . The region between $P = 0$ and $P = 1$ is stable.



(c) The case in which the effective potential V_{eff} is constant at 1 . The region between $P = 1$ and $P = 0$ is stable.

Figure 3: The case in which the effective potential V_{eff} is constant.

The shape of the orbit usually depends on the nature of the effective potential. Therefore, it is natural that the orbit will have several dependencies on both μ and μ_0 . The unstable potential V_{eff} in the potential plane will be obtained if the condition $\mu_0 = 0$ is imposed. We plot V_{eff} versus μ_0 with $\mu = 1$ and $\mu_0 = 0$ in Figure 4. The region between $P = 0$ and $P = 1$ is stable.

2. ROTATION ORBIT AND BRAGG PEAK SPREAD

To illustrate the behavior of the structure factor calculated in this study, the following two different configurations are plotted with the help of eq. (5) $\text{with } \lambda = 0.11 \text{ nm}$, i.e. $\frac{2\pi \sin \theta}{\lambda} = \frac{2\pi \sin \theta}{0.11 \text{ nm}}$ and are presented in a two-dimensional plot. The values of the structure factor will depend on various parameters like θ . In this study, the Bragg angle θ is fixed at 20° for both plots.

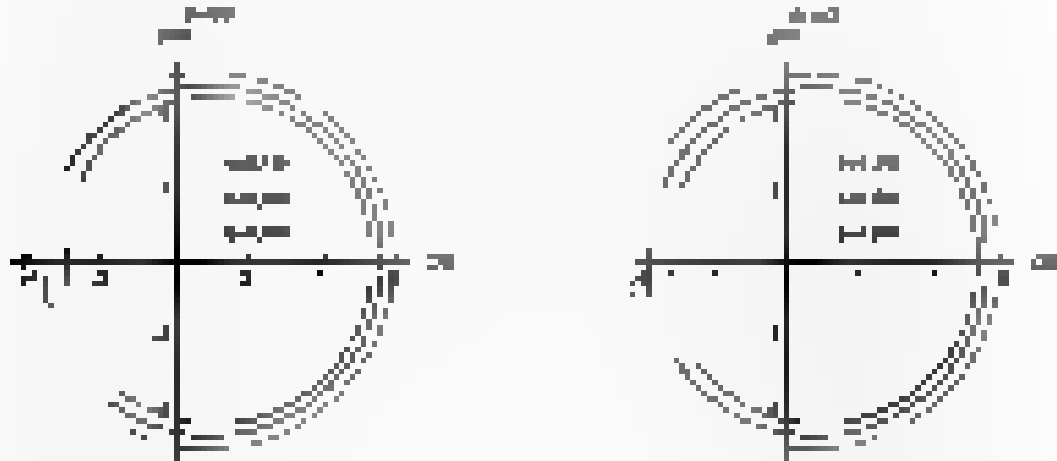


Figure 3. Variation of structure factor F with $m=2$ and F with $m=3$ and $n=2$

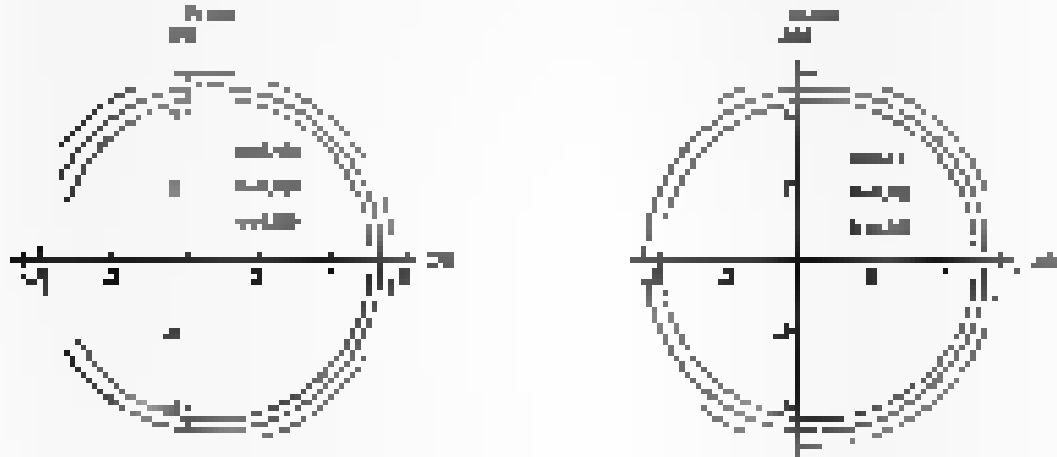


Figure 4. Variation of structure factor F with $m=2$ and F with $m=3$ and $n=3$

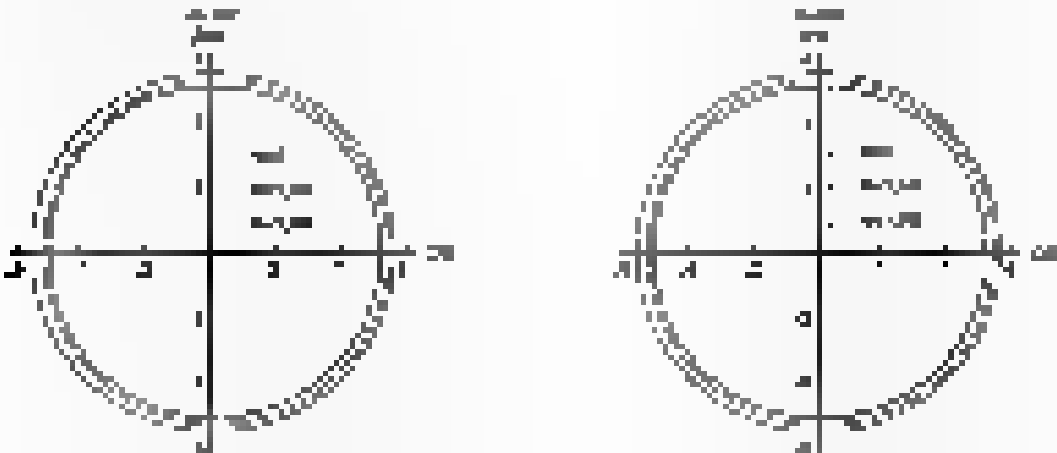


Figure 5. Variation of structure factor F with $m=2$ and F with $m=3$ and $n=4$

THE DEVIATION FROM THE CIRCULAR BEAM OF THE SHADOW α_1 AND THE SIZE β_1 OF THE SHADOW

Using Fig. 26, it can be shown that the size of the shadow decreases with the increase in the value α_1 . Here, determination of the shape of the shadow and of the lateral points by means of mathematical expressions will be done more simply and with fewer calculations if we use Fig. 27 (Fig. 24 and Fig. 25 B show the lateral points α_1 and β_1 and of the shadow that coincide the lateral points α_1 and β_1 of the shadow with the right calculations of the shape of the shadow have been found previously in all cases.

3.3 The deviation from the circular beam of the shadow α_1 and the size β_1 of the shadow

Using the parameters which are introduced in Fig. 24 and Fig. 25 B, let us find the deviation from the circular beam of the shadow α_1 and the size β_1 of the shadow (Fig. 27) and let

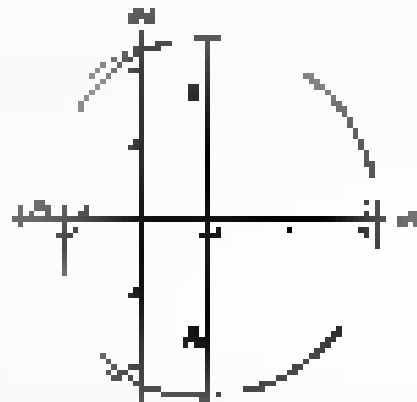


Fig. 27. A sketch of the field of the shadow that the value α_1 which is the deviation from the circular beam of the shadow and the maximum value of the size β_1 .

For calculating these parameters, we consider five points. The four points (α_1, β_1) , (α_2, β_2) , (α_3, β_3) and (α_4, β_4) are, respectively, the top, bottom, rightmost, and the leftmost points of the shadow and (α_5, β_5) is the leftmost point of the reference circle. From simple geometry we find the expressions for the values of α_1 given in Fig. 27 B) and the deviation from the circular beam β_1 given in Fig. 27 B) by using Fig. 27 and the various expressions for all subsequent steps, we have $\beta_1 = \beta_5 - \alpha_1$.



Fig. 28. The diagram of the deviation from the circular beam of the shadow α_1 and the size β_1 of the shadow. α_1 is in degrees and β_1 is in mm. The figure on the right represents the maximum value β_1 and the deviation from the circular beam α_1 for all values of h from 0 to 10 mm and β_1 is in mm.

THE DIFFERENCE BETWEEN THE DISTRIBUTION OF THE CHANGES Δx_{ij} WITH THE SAME
 TYPE OF THE QUANTITY

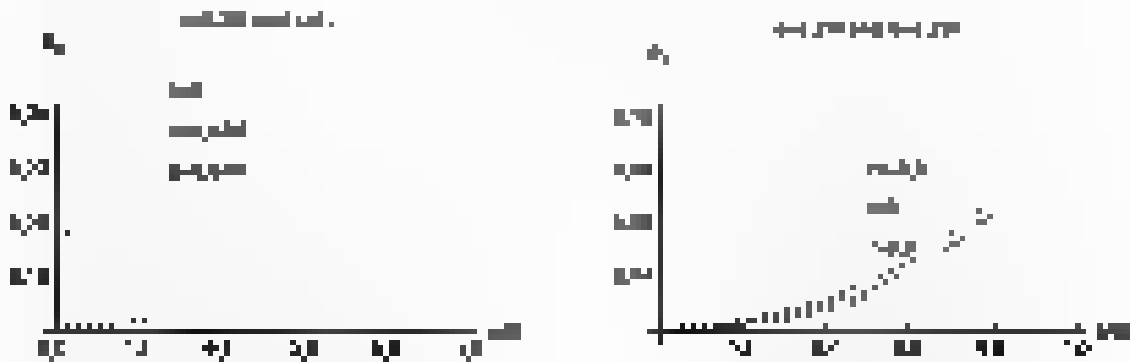


Figure 18 The figure shows the distribution of changes Δx with the same type of the quantity Δx and Δx . The right one corresponds to changes of Δx with the different values of Δx and Δx .



Figure 19 The figure shows the distribution of changes Δx with the same type of the quantity Δx and Δx . The right one corresponds to changes of Δx with the different values of Δx and Δx .



Figure 20 The figure shows the distribution of changes Δx with the same type of the quantity Δx and Δx . The right one corresponds to changes of Δx with the different values of Δx and Δx .

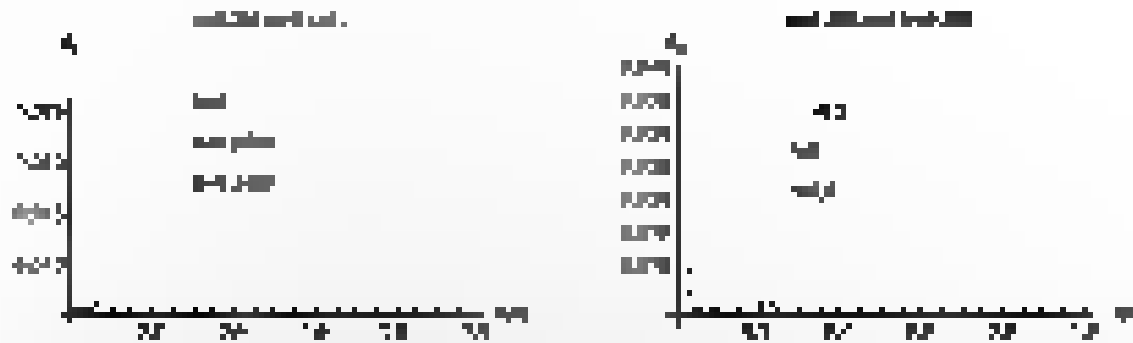


Figure 3. (L) The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations. The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations. The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations.

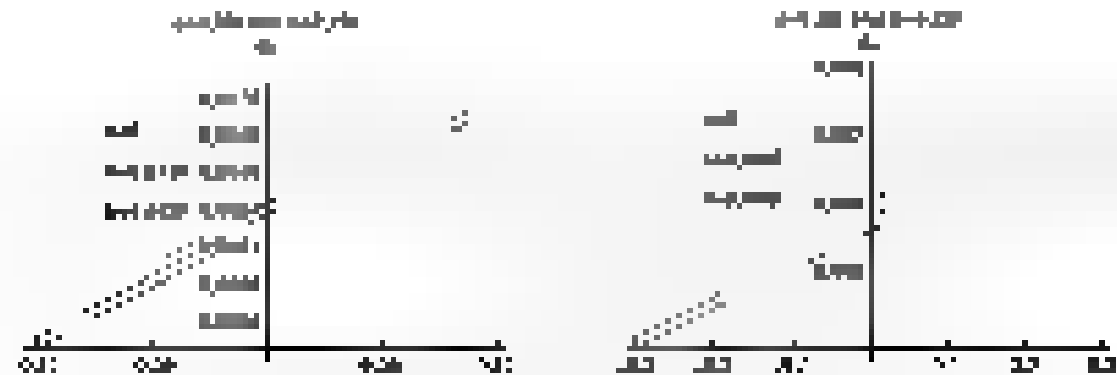


Figure 4. The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations. The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations. The logarithmic black hole mass represents the mass (d_b) of black holes in the red, blue, and grey populations.

Here, we have given an absolute value, the deviation of the average r_p of the black hole domain from the average r_p of the black hole domain. The deviation of the average r_p of the black hole domain from the average r_p of the black hole domain is given by

Table 1. Deviation r_p for the black hole domain with $r_p = 0.1$ and $r_p = 0.2$.

	0.1	0.2	0.3
$r_p = 0.1$	0.0000000	0.0000000	0.0000000
$r_p = 0.2$	0.0000000	0.0000000	0.0000000
$r_p = 0.3$	0.0000000	0.0000000	0.0000000

Table 1. Deviation r_p for the black hole domain with $r_p = 0.1$ and $r_p = 0.2$. The deviation of the average r_p of the black hole domain from the average r_p of the black hole domain is given by

5.1 The black hole energy conversion

It is well known that all black holes have a minimum size, the minimum size of the black hole domain, and the process continues until the black hole collapses down to a point. In the standard cosmological model, the black hole domain of radius r_p has a minimum size r_p . Here we can use the energy conversion process to explain the evolution from the black hole domain to the black hole domain. The energy conversion process is related to the frequency ν given by Eq. (12).

The following distribution is plotted in Fig. 15.14

$$\tau_h = \frac{y}{\sqrt{1+y^2}} \frac{2\sqrt{1+y^2} - 1 - \frac{1}{y^2} + 1}{2\sqrt{1+y^2} + \frac{1}{y^2} + 1} \quad (15.14)$$

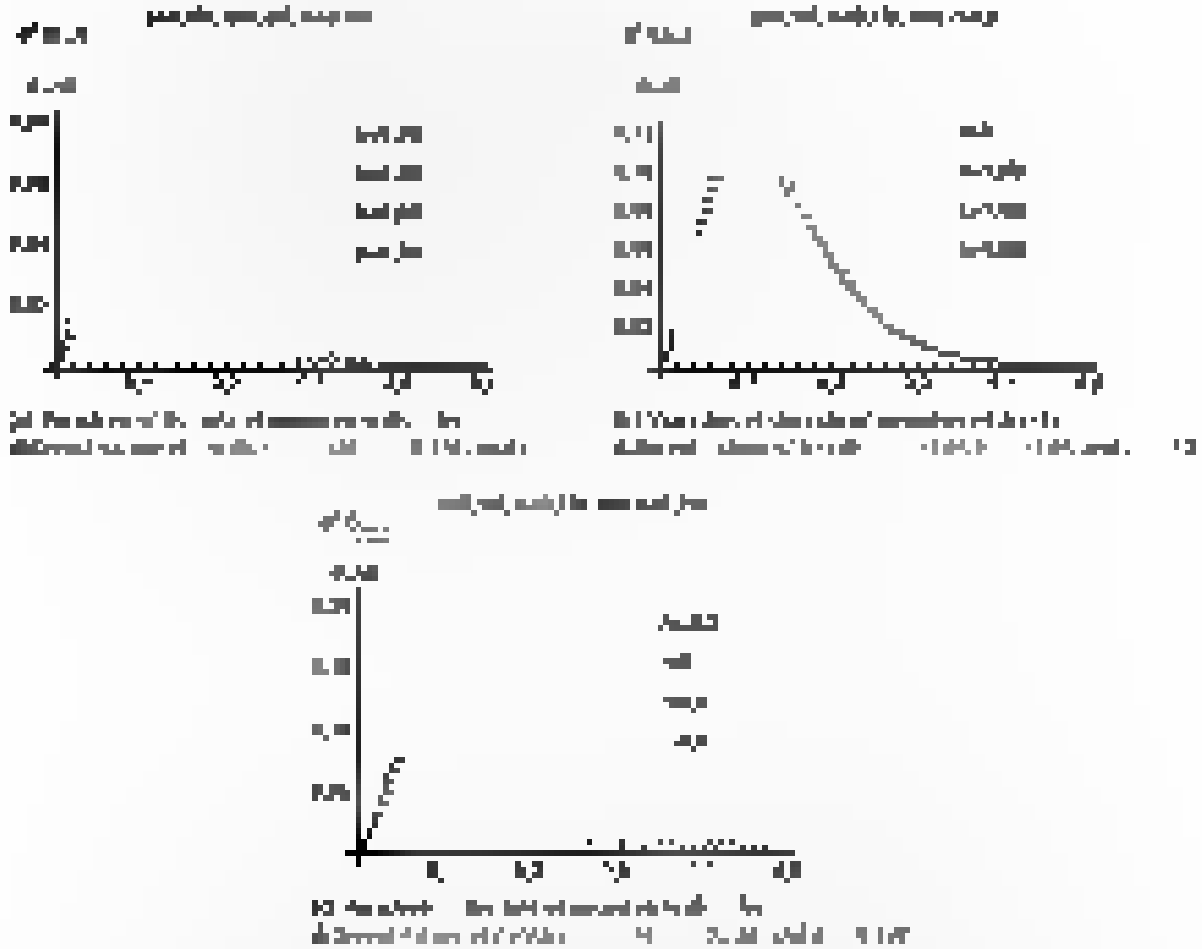


Figure 15.14: The ratio of masses of members of the cluster

In Fig. 15.15, we have studied from the energy distribution, and will obtain with the variation of the parameters τ_h and τ_h it is observed that the ratio of masses is higher for smaller values of τ_h . However, the reason is the reason is more complex. In order to obtain the reason, we have to consider the energy distribution of the cluster. The energy distribution is related to the black hole, which is a point source with the Kerr-Newman metric. From the plot, we can see that for negative τ_h , there is an accumulation of black hole at the center of the cluster, which is a redshift in the cluster of redshift.

Chapter 6

Answers

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
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57. $\frac{1}{2}$
58. $\frac{1}{2}$
59. $\frac{1}{2}$
60. $\frac{1}{2}$

ALTERNATIVE SCHEMES OF THE COMMON EUROPEAN CONVENTION

DATE: 10.11.2014

where α is a positive real number that measures a literature's effectiveness at the end of the review and β is the effective parameter of the literature that is subject to comparison. $\beta = 1$ is the average period of the literature and is equal to 1 in the end. The period is an average of the literature. The literature is

Dr. L. J. ...

where \mathbf{A} is the $n \times n$ matrix and \mathbf{b} is the $n \times 1$ vector defined as

$$k = k_{\text{max}} \left(1 - \frac{v}{v_{\text{max}}} \right) \quad \text{for} \quad \frac{v}{v_{\text{max}}} \leq 1 \quad (11)$$

Notes that no particular focus was revealed about the low attendance people usually attend services and is spread evenly. However the program is open to the community people, and is open to the low attendance people.

Because the performance of the quadratic model has not been formally evaluated in the literature, we have conducted a series of experiments to evaluate the quadratic model. In the experiments, we compare the quadratic model with the following models:

$$A + \frac{B}{C} = \frac{AC + B}{C} \quad \text{والا}$$

[illegible]

$$H_{\text{eff}} = \frac{dK}{2} + \frac{dF}{2} \quad (1)$$

and the Airman was already much older

$$A_{\mu} = K \frac{d^2 \psi}{d\tau^2} \quad A_{\mu} = \frac{d^2 \psi}{d\tau^2} \quad \text{[1]}$$

between edge - material density - velocity of longitudinal - pressure - relationship due to interaction corresponding to volume with no expansion of - since the motions are linearly independent - thus, the modulus - related to between - available to reduce with the effective amount of volume - total in the physical sense - associated with the flow - wave state of the process [11]. Thus, the existing condition - corresponding to a mechanism - the computed and measured wave - flow and energy of - to the response of material results.

דף	שם	מחבר	מספר
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A useful decomposition for \mathcal{D}^2 is $\mathcal{D}^2 = \mathcal{D}^2_{\text{diag}} + \mathcal{D}^2_{\text{off-diag}}$. The scalar part will be exponentially simplified, while the bilinear part $\mathcal{D}^2_{\text{off-diag}}$ needs to be dealt with the attention.

4.2. Asymptotic analysis of Σ_j for asymptotically large

It is straightforward to express the total equation of Σ_j in the following form

$$\frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) - \frac{Q_j}{x^2} - \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.1)$$

where $R_{j+1}(x)$ is the solution for the term and the two terms and Q_j is the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term.

$$\frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) - \frac{Q_j}{x^2} - \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.2)$$

where the asymptotic analysis of $R_{j+1}(x)$ is performed by using the asymptotic analysis for the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term.

$$R_{j+1}(x) = \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) + \frac{Q_j}{x^2} + \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.3)$$

where the asymptotic analysis of the term is performed by using the asymptotic analysis for the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term.

$$R_{j+1}(x) = \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) + \frac{Q_j}{x^2} + \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.4)$$

where the asymptotic analysis of the term is performed by using the asymptotic analysis for the term.

$$R_{j+1}(x) = \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) + \frac{Q_j}{x^2} + \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.5)$$

where the asymptotic analysis of the term is performed by using the asymptotic analysis for the term.

$$R_{j+1}(x) = \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) + \frac{Q_j}{x^2} + \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.6)$$

where the asymptotic analysis of the term is performed by using the asymptotic analysis for the term. The asymptotic analysis is performed by expanding the solution for large j in powers of j and by using the asymptotic analysis for the term.

$$R_{j+1}(x) = \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) + \frac{Q_j}{x^2} + \frac{d^2}{dx^2} \left(\frac{R_{j+1}(x)}{x^2} \right) = 0, \quad (4.7)$$

For the following we obtain the upper solution $p + \epsilon$ and $\mu^2 = 0$. We only drop all the terms except those which describe the two main small resonances, and that yields equations (9.11)

$$i\frac{dA_{\pm}(p)}{dp} + \frac{2\alpha B_{\pm}(p)}{p} = 0, \quad \frac{d\mu}{dp} + \frac{1}{p} \mu = 0, \quad B_{\pm}(p) = \quad (9.11)$$

where $\mu = \frac{d\sqrt{1-\mu^2}}{d\sqrt{1-\mu^2}}$. Equation (9.11) has the general solution

$$\begin{aligned} A_{\pm}(p) &= \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \\ &+ \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.12) \end{aligned}$$

Here $A_{\pm}(p)$ stands for the asymptotic behaviour of $A_{\pm}(p)$ and $B_{\pm}(p)$ in order to match the solution with (9.11) we consider the small p behaviour of the solution (9.11). For small $p \rightarrow 0$, the equations (9.11) read as

$$i\frac{dA_{\pm}(p)}{dp} + \frac{2\alpha B_{\pm}(p)}{p} = 0, \quad \frac{d\mu}{dp} + \frac{1}{p} \mu = 0, \quad B_{\pm}(p) = \quad (9.13)$$

The solution (9.11) and (9.13) are compatible for matching with those two main resonances of interest. The matching at the asymptotic limits is (9.11) and (9.13) enable us to compare the results from the solution (9.11) and (9.13).

$$\begin{aligned} &= \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \\ &+ \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.14) \end{aligned}$$

We expand equation (9.11) around $p \rightarrow 0$ which after expansion yields

$$\begin{aligned} &= \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \\ &+ \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{\sqrt{1-\mu^2}} \frac{1}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.15) \end{aligned}$$

With the approximation $p \rightarrow 0$, $\mu \rightarrow 0$, $\frac{d\mu}{dp} \rightarrow 0$ and $\frac{dA_{\pm}(p)}{dp} \rightarrow 0$ we get the asymptotic solution (9.11) for small $p \rightarrow 0$

$$A_{\pm}(p) = \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.16)$$

and

$$\begin{aligned} &= \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \\ &+ \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.17) \end{aligned}$$

and

$$\begin{aligned} &= \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \\ &+ \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \frac{1}{p} \frac{\sqrt{1-\mu^2}}{\sqrt{1-\mu^2}} \left(\frac{1}{\mu} \right) + \quad (9.18) \end{aligned}$$

Substituting the superstrain value of η and η_0 from Eq. (1) into the above equations we have [24]

$$\begin{aligned} P_{11}^0 &= A \left(1 - \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} \right) + P_0 \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2}} \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} \quad (23) \\ P_{12} &= 2\sqrt{1-\eta_0^2} \eta_0 \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} + A \left(1 - \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} \right) \\ P_{22} &= \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2}} \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} + P_0 \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} \end{aligned}$$

and

$$\begin{aligned} P_{11}^0 &= A \left(1 - \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} \right) + P_0 \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2}} \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} \quad (24) \\ P_{12} &= 2\sqrt{1-\eta_0^2} \eta_0 \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} + \frac{1}{\eta_0^2} \frac{\sqrt{1-\eta_0^2}}{\eta_0^2} \\ P_{22} &= \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2}} \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} + P_0 \frac{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}}{\sqrt{1-\eta_0^2+2\eta_0^2(1+\eta_0^2)}} \end{aligned}$$

The amplification factor amplification number can be

$$A_{\omega} = \frac{|P_{11}^0|}{P_{11}^0} \quad (25)$$

Equation (25) is given in terms of the amplification factor obtained by making use of the asymptotic method used. When $\frac{P_{11}^0}{P_{11}^0}$ acquires a value greater than unity there will be a gain in amplification factor that corresponds to superstrain phenomena. However, a negative value of the amplification factor indicates a loss that corresponds to the anti-superstrain of superstrain. To show the effect of superstrain on the superstrain phenomena, we will consider the case $\eta_0 = 0.5$ (the difference between the positive and negative values of η_0 is not important for the leading multipole $\eta_0 = 0$ and taking different values of η_0 gives rise and polarity of the tensor values parameter. From the Fig. 1 it is seen that the effect of the superstrain on the amplification factor is not significant when the superstrain value is not too high, i.e. $\eta_0 = 0.5$.

For negative η_0 superstrain factor takes negative value which shows the loss of amplification of superstrain. The plot also shows superstrain factor with the decrease of the value of η_0 (superstrain parameter) for superstrain phenomena increases and the increase of the value of the value of the tensor values parameter decreases. In Fig. 1 (a) it is seen that the effect of the parameter η_0 on the superstrain phenomena is not significant when the superstrain value is not too high, i.e. $\eta_0 = 0.5$. However, in the case of $\eta_0 = 0.5$ the superstrain effect increases as η_0 increases from Fig. 1 (b).

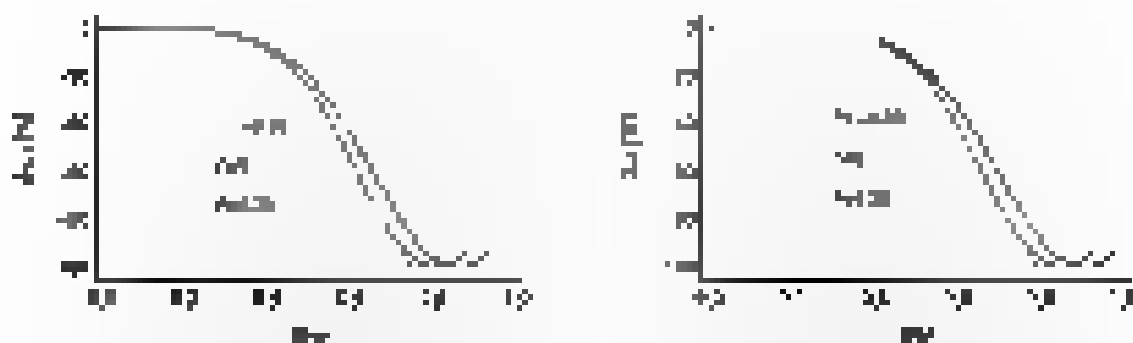


Figure 11: Variable and sample size for the number of the corresponding model and the value of R_0 and R_1 for $R_2 = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$.

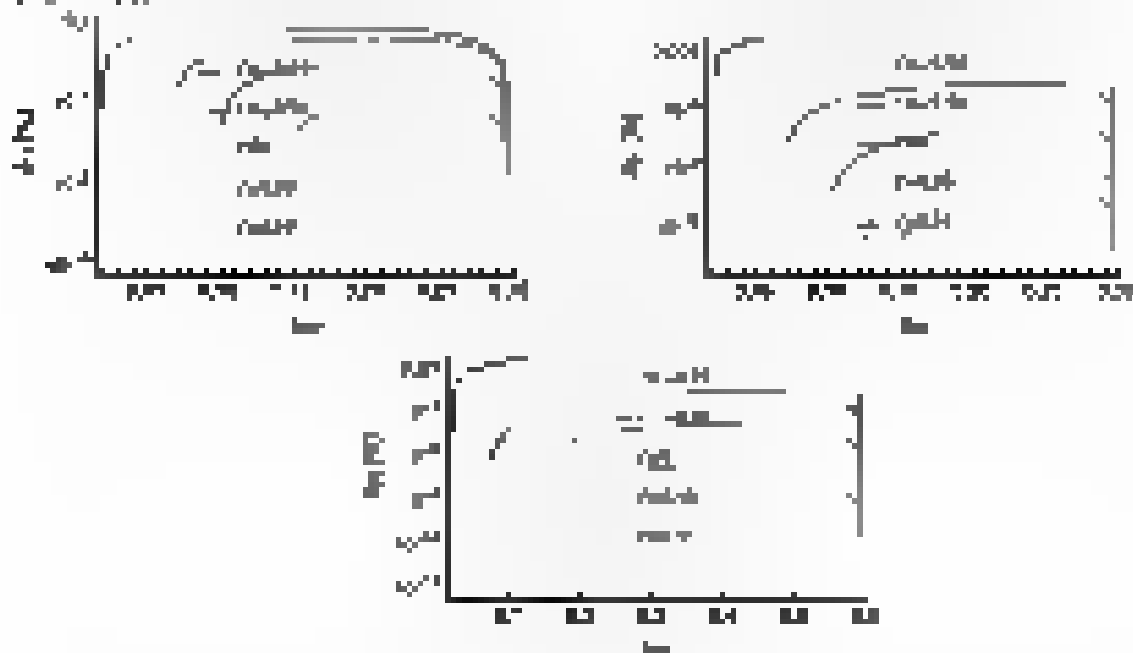


Figure 12: Variable and sample size for the number of the corresponding model and the value of R_0 and R_1 for $R_2 = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 4.5, 5.0$.

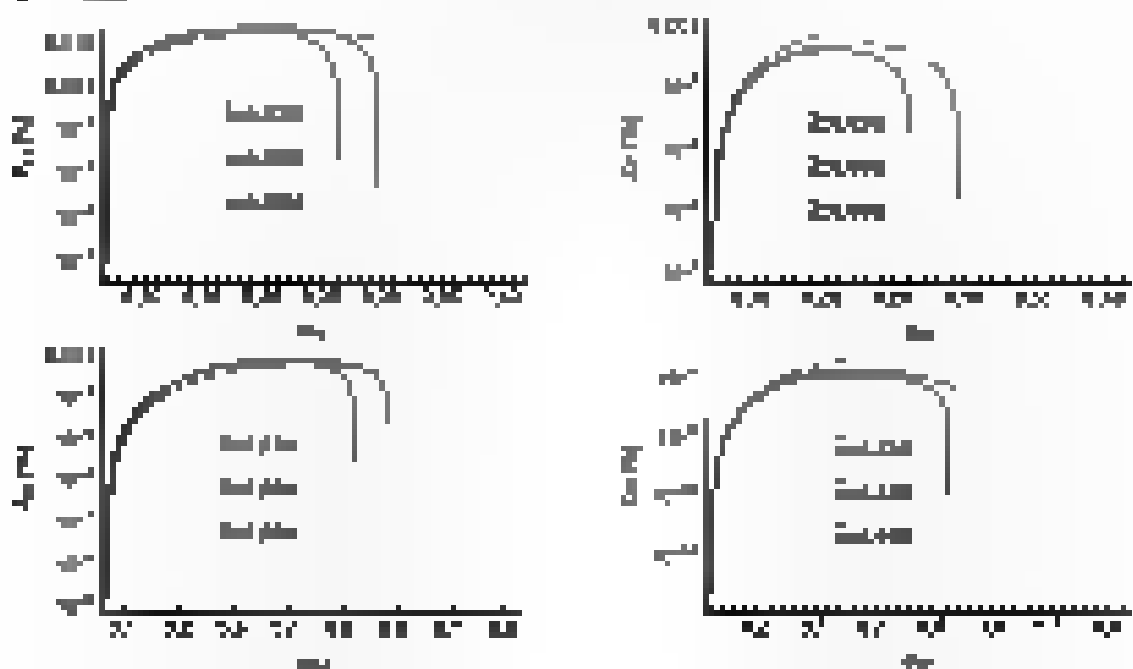


Figure 13: Variable and sample size for the number of the corresponding model and the value of R_0 and R_1 for $R_2 = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 4.5, 5.0, 6.0, 6.5, 7.0$.

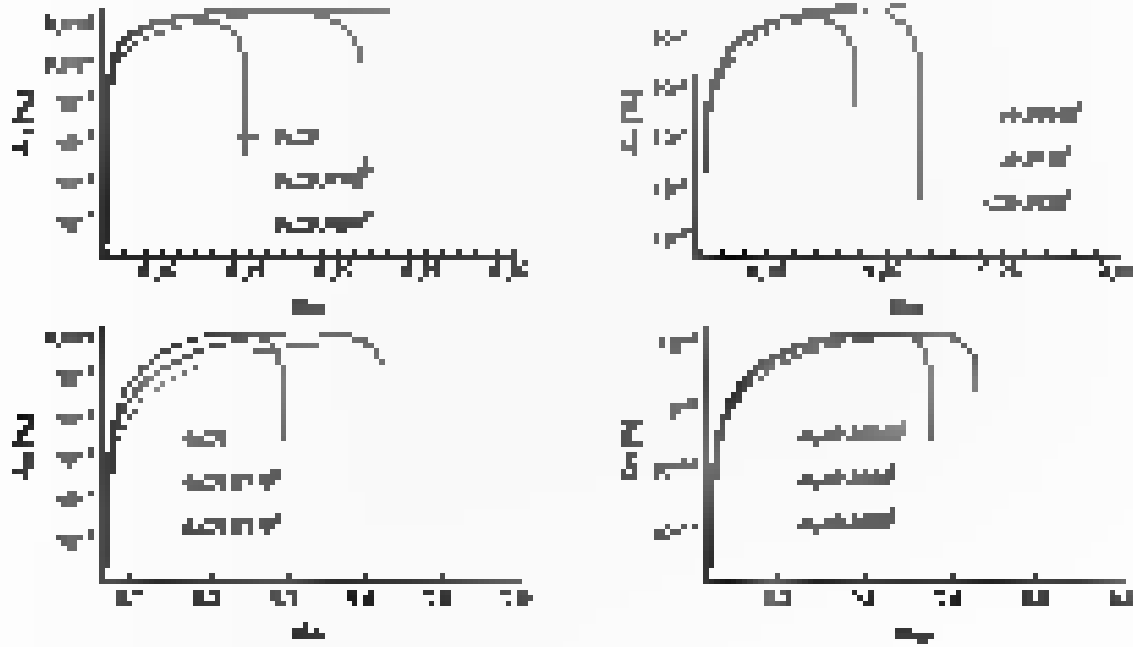


FIGURE 14. Stability of superadiant instabilities with β in various multipole modes $\mu = 0.1$ and $\alpha = 0.1, 0.2, 0.3$ for $\mu = 0.1$ and $\beta = 0.1, 0.2, 0.3, 0.4$.

4.3 Superadiant instability for Lorentz-viaiviere and non-coaxiative Kerr-like black hole

From equation (3.5) we have

$$\Delta_{\text{eff}} = \alpha \frac{\partial \mathcal{E}_{\text{eff}}}{\partial \omega} - T \mathcal{E}_{\text{eff}} = 0 \quad (3.51)$$

where for slowly rotating black hole ($\mu = 0.1$)

$$\mathcal{E} = \left(\frac{1}{r} - \frac{\gamma_{\text{eff}}}{r^2} - \frac{\omega_{\text{eff}}^2}{r^2} \right) \Delta_{\text{eff}} = 0 \quad (3.52)$$

If we now look for the black hole linear instabilities, we would have the following solutions to the radial equation (3.24)

$$\mathcal{E}_{\text{eff}} = \frac{1}{\sqrt{r}} e^{i\omega_{\text{eff}} t} = \frac{1}{\sqrt{r}} e^{i\omega_{\text{eff}} t} \quad (3.53)$$

The above solution represents the charge perturbation equation that the perturbation is a black hole linear instability equation which is a small perturbation to the background solution provided that $\gamma < 1$. With the radial coordinate

$$r_{\text{eff}} = \sqrt{r} \mathcal{E}_{\text{eff}} \quad (3.54)$$

the radial equation (3.24) can be

$$\frac{d^2 r_{\text{eff}}}{dr_{\text{eff}}^2} + \left(\omega_{\text{eff}}^2 - \frac{1}{r_{\text{eff}}^2} \right) r_{\text{eff}} = 0 \quad (3.55)$$

with

6. STATIONARY INSTABILITY FOR LORENTZ VOLTAGE AND NON-COLOCATED PRESSURE AND CURRENT SOURCES

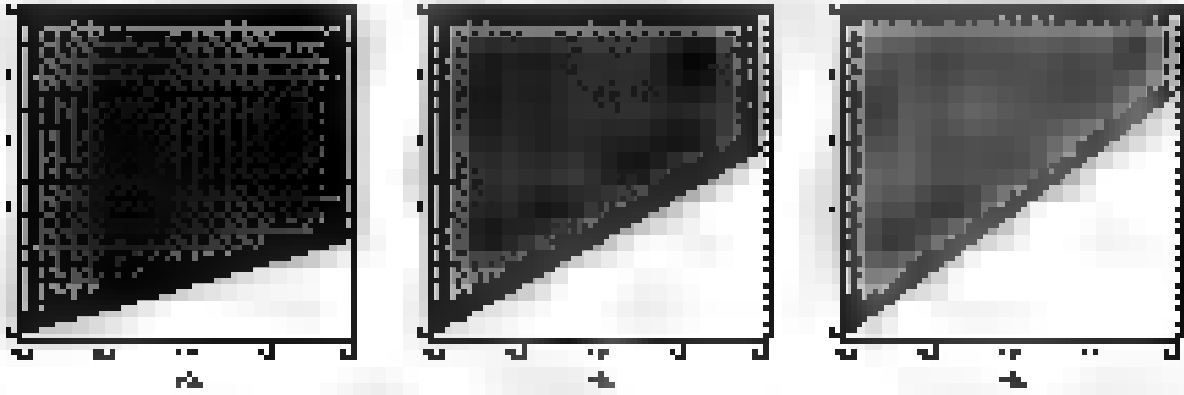


Fig. 13 Parameter plots: (a) Lorentz voltage and non-colocated pressure and current sources; (b) Lorentz voltage and non-colocated current source; (c) Lorentz voltage and non-colocated pressure source. $\alpha = 0.1$ and $\beta = 0.1$.

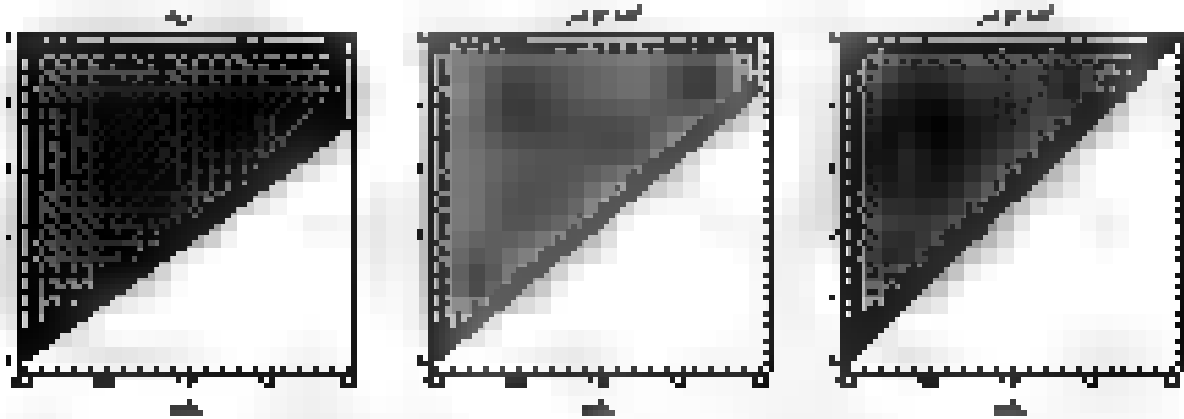


Fig. 14 Parameter plots: (a) Lorentz voltage and non-colocated pressure and current sources; (b) Lorentz voltage and non-colocated current source; (c) Lorentz voltage and non-colocated pressure source. $\alpha = 0.1$ and $\beta = 0.1$.

$$\mathcal{P} = \mathcal{C} + \frac{\mathcal{U}(\omega = \frac{\omega_0 \sqrt{1-\alpha^2}}{\sqrt{1-\beta^2}})}{\frac{\partial \mathcal{U}}{\partial \omega}} \quad (6.54)$$

where $\mathcal{U}(\omega)$ represents the voltage. By denoting the vector $\mathbf{U} = [U]$, the impedance term of the voltage potential (6.54) can be

$$\mathcal{U}(\omega) = \mu^2 \mathbf{U} + \mathbf{U} \cdot (\mathbf{I} + \mathbf{J}) \frac{1+\alpha^2}{1-\alpha^2} + \mathbf{U} \cdot \mathbf{J} \frac{1+\beta^2}{1-\beta^2} \quad (6.55)$$

To obtain the trapping instability by the above voltage potential (6.55) due to the impedance (6.54) at zeroth order ($\alpha = 0$ and $\beta = 0$) [60]. The voltage term in the above expression corresponds to the voltage source at the origin, which is the case of the voltage [31]

$$\frac{\partial}{\partial \omega} \mathcal{U}(\omega) = \mathcal{U}(\omega) \quad (6.56)$$

which the integrated space of the voltage source has been shown below. The other terms in the above expression represented stability, because the black hole horizon. The structure of the voltage term of (6.54) is non-singular. For the black hole well, the voltage source is $\mathcal{U}(\omega) = \mathcal{U}(\omega_0)$.

Let $\mathcal{A}_\mu^{\text{ref}}$ represent the amplitude of the incoming wave, where $\mathcal{A}_\mu^{\text{ref}}$ is the corresponding quantity of the incoming wave with $\mathbf{a} = \mathbf{a}^{\text{ref}}$. Along with these, the amplitude of the reflected part of the wave is $\mathcal{A}_\mu^{\text{refl}}$ with $\mathbf{a} = \mathbf{a}^{\text{refl}}$.

We now compute the Wronskian for the region \mathcal{R} as the difference of the Wronskian \mathcal{W} of $\mathcal{A}_\mu^{\text{ref}}$ and $\mathcal{A}_\mu^{\text{refl}}$ in this region as

$$W_\mu = W_{\mathcal{A}_\mu^{\text{ref}}} - \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} - W_{\mathcal{A}_\mu^{\text{refl}}} - \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \quad (4.41)$$

and the Wronskian in the ray mode

$$W_\mu = W_{\mathcal{A}_\mu^{\text{ref}}} - \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} - W_{\mathcal{A}_\mu^{\text{refl}}} - \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \quad (4.42)$$

The relative error is easily estimated. Our knowledge of standard theory of elliptic functions, but makes it can be inferred that these $\mathcal{A}_\mu^{\text{ref}}$ and $\mathcal{A}_\mu^{\text{refl}}$ are estimated at $\sim 10^{-10}$. Thus, the Wronskian W_μ of the region \mathcal{R} is equal to the difference of the Wronskian $W_{\mathcal{A}_\mu^{\text{ref}}}$ and $W_{\mathcal{A}_\mu^{\text{refl}}}$. Thus, the relative error of the Wronskian W_μ is the relative error of the Wronskian $W_{\mathcal{A}_\mu^{\text{ref}}}$ and $W_{\mathcal{A}_\mu^{\text{refl}}}$ in the region \mathcal{R} .

$$\mathcal{A}_\mu^{\text{refl}} = \mathcal{A}_\mu^{\text{ref}} - \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \quad (4.43)$$

The above equation implies that if $\frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} = 0$, then $\mathcal{A}_\mu^{\text{refl}} = \mathcal{A}_\mu^{\text{ref}}$ and the Wronskian W_μ is equal to the Wronskian $W_{\mathcal{A}_\mu^{\text{ref}}}$. The relative error of the Wronskian W_μ is the relative error of the Wronskian $W_{\mathcal{A}_\mu^{\text{ref}}}$ in the region \mathcal{R} .

4.5 Calculation of the amplification factor \mathcal{Z}_μ for superelliptic

We now calculate the Wronskian W_μ as

$$\begin{aligned} & \mathcal{A}_\mu^{\text{ref}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \mathcal{A}_\mu^{\text{refl}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \\ & + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \end{aligned} \quad (4.44)$$

We will derive the Wronskian and the Wronskian in the region \mathcal{R} as a function of the Wronskian W_μ and the Wronskian in the region \mathcal{R} as a function of the Wronskian W_μ . We apply the change of variable $z = \frac{1}{\mathcal{A}_\mu^{\text{ref}}}$. Along this change, the Wronskian W_μ is equal to W_μ under the approximation $\mathcal{A}_\mu^{\text{ref}} \approx 1$.

$$\begin{aligned} & \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \\ & = \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \\ & = \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \end{aligned} \quad (4.45)$$

where $\mathcal{A}_\mu^{\text{ref}} = 1$ and $\mathcal{A}_\mu^{\text{refl}} = \frac{1}{\mathcal{A}_\mu^{\text{ref}}}$. For the region \mathcal{R} , we have $\mathcal{A}_\mu^{\text{ref}} = 1$ and $\mathcal{A}_\mu^{\text{refl}} = \frac{1}{\mathcal{A}_\mu^{\text{ref}}}$ and hence the above equation reduces to

$$\begin{aligned} & \mathcal{A}_\mu^{\text{ref}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \mathcal{A}_\mu^{\text{refl}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \\ & = \mathcal{A}_\mu^{\text{ref}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \mathcal{A}_\mu^{\text{refl}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} + \frac{1}{\mathcal{A}_\mu^{\text{ref}}} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} - \frac{1}{\mathcal{A}_\mu^{\text{refl}}} \frac{d\mathcal{A}_\mu^{\text{refl}}}{dz} \frac{d\mathcal{A}_\mu^{\text{ref}}}{dz} \end{aligned} \quad (4.46)$$

The approximation $\mathcal{A}_\mu^{\text{ref}} \approx 1$ is obtained from the consideration that the region \mathcal{R} is a region of the complex plane. The approximation $\mathcal{A}_\mu^{\text{refl}} \approx \frac{1}{\mathcal{A}_\mu^{\text{ref}}}$ is obtained from the consideration that the region \mathcal{R} is a region of the complex plane.

1.5 CALCULATION OF THE AMPLIFICATION FACTOR A_v FOR SUBTHRESHOLD

The general solution of above equation is in terms of hyperbolic functions. The first term in (33) can be written as:

$$R_{eq}(s) = -\frac{g_m \sqrt{1+\beta}}{\sqrt{1+\beta+\beta^2}} (1 - \beta s) \quad (35)$$

Using this in (33) we get

$$V_o(s) = \frac{1}{\sqrt{1+\beta}} \frac{1}{s^2 + \frac{1+\beta}{\tau_p} s + \frac{1+\beta}{\tau_p^2}} \cdot \frac{1}{s} \quad (36)$$

It is noted that (36) is the Laplace transform of constant hyperbolic cosine function $F_1(s) = 1$

$$F_{1,inv}(s) = \frac{1}{F_1(s)} = \frac{1}{1} = 1 \quad (37)$$

The inverse Laplace transform of the resulting function of the denominator where the solutions exist in the right half plane is obtained by using the hyperbolic cosine of the above expression. The eq. (36) for large β can be made

$$V_o(s) = \frac{1}{\sqrt{1+\beta}} \frac{1}{s^2 + \frac{1+\beta}{\tau_p} s + \frac{1+\beta}{\tau_p^2}} \cdot \frac{1}{s} \quad (38)$$

For brevity we can use the approximation $\frac{1}{\sqrt{1+\beta}} \approx \frac{1}{\sqrt{\beta}}$. We may drop all the terms except those which describe the low-frequency behaviour and the resulting equation of (38) can

$$\frac{d^2 V_o(s)}{ds^2} + \frac{dV_o(s)}{ds} = \frac{1}{\beta} \frac{1}{s^2} \quad (39)$$

where $\frac{1}{\beta} = \frac{1}{\sqrt{1+\beta}} \approx \frac{1}{\sqrt{\beta}}$ by using (37) = the proof is clear

$$V_o(s) = \frac{1}{\sqrt{1+\beta}} \frac{1}{s^2 + \frac{1+\beta}{\tau_p} s + \frac{1+\beta}{\tau_p^2}} \cdot \frac{1}{s} \quad (40)$$

where $\frac{1}{\sqrt{1+\beta}}$ is taken as the first Laplace function. In order to get the particularity of (39) we require the overall characteristic of the solution of (39). For this, let $\beta \rightarrow \infty$, the equation (39) takes the form

$$V_o(s) = \frac{1}{\sqrt{1+\beta}} \frac{1}{s^2 + \frac{1+\beta}{\tau_p} s + \frac{1+\beta}{\tau_p^2}} \cdot \frac{1}{s} \quad (41)$$

The solution of (39) and (41) has complex roots of poles and are responsible for oscillating. The increasing of the asymptotic solution (39) and (41) can be up to compare the ratio with that of (39)

$$A_v = \frac{1}{\sqrt{1+\beta}} \frac{1}{s^2 + \frac{1+\beta}{\tau_p} s + \frac{1+\beta}{\tau_p^2}} \cdot \frac{1}{s} \quad (42)$$

1.5 CALCULATION OF THE AMPLIFICATION FACTOR A_v FOR SUBTHRESHOLD NOISE

and

$$A_{v1} = \frac{g_m \left(1 - \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right)}{g_{m1} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right) + g_{m2} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right) + g_{m3} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right)} \quad (1.47)$$

$$A_{v2} = \frac{g_{m2} \left(1 - \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right)}{g_{m1} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right) + g_{m2} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right) + g_{m3} \left(1 + \frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}} \right)} \quad (1.48)$$

The amplification factor is given by

$$A_{vm} = \frac{A_{v1} + A_{v2}}{2} \quad (1.49)$$

Equation (1.47) is a general expression of amplification factor obtained making use of the asymptotic matching method. When $\frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}}$ is equal to unity there will be gain in amplification. If $\frac{g_{m1} g_{m2}}{g_{m1} g_{m2} + g_{m1} g_{m3} + g_{m2} g_{m3}}$ is less than unity there will be loss in amplification. However, when the amplification factor is negative indicates loss, which corresponds to the loss of all kinds of noise reduction.

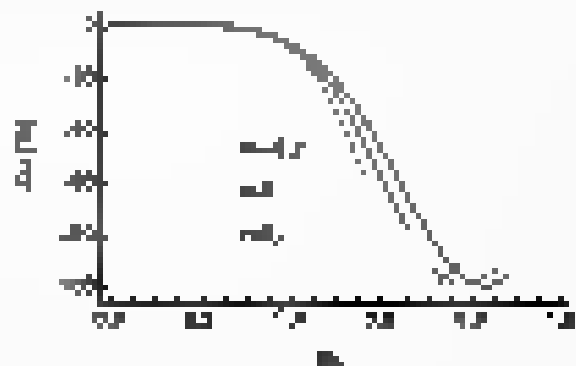
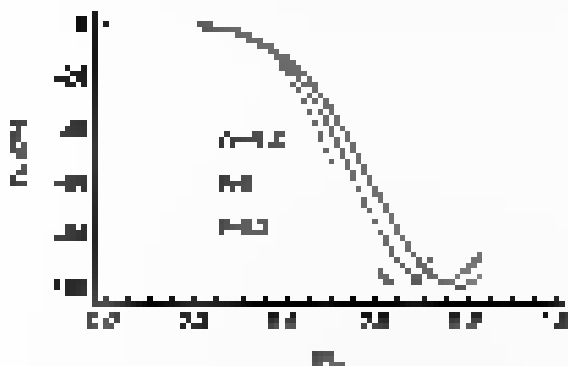


Fig. 1.47. Asymptotic of amplification factor with the use of asymptotic matching with

and 1.48

(1.49)

15. CALCULATION OF THE AMPLIFICATION FACTOR Δ_{eff} FOR SUBSTRATE INHIBITION

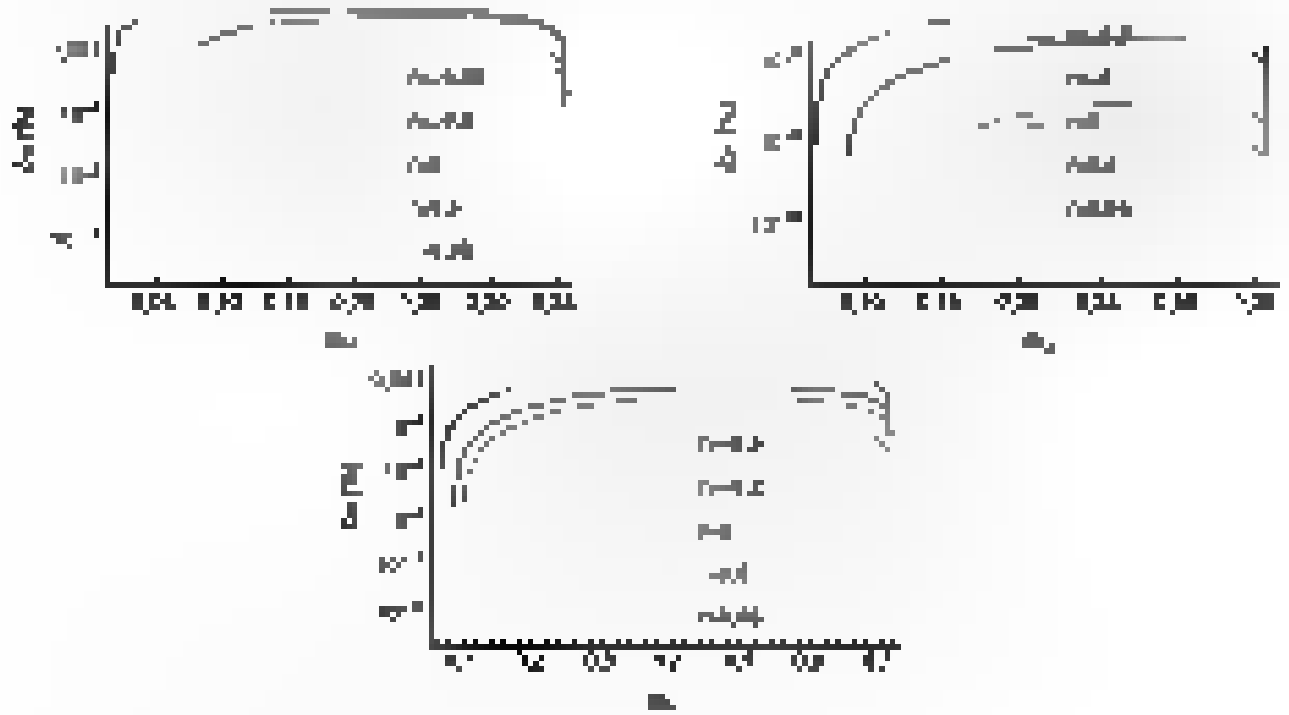


Fig. 10. - Variation of amplification factor Δ_{eff} with substrate concentration S_0 for $r=0.2$ (top left), $r=0.5$ (top right), and $r=0$ (bottom center).

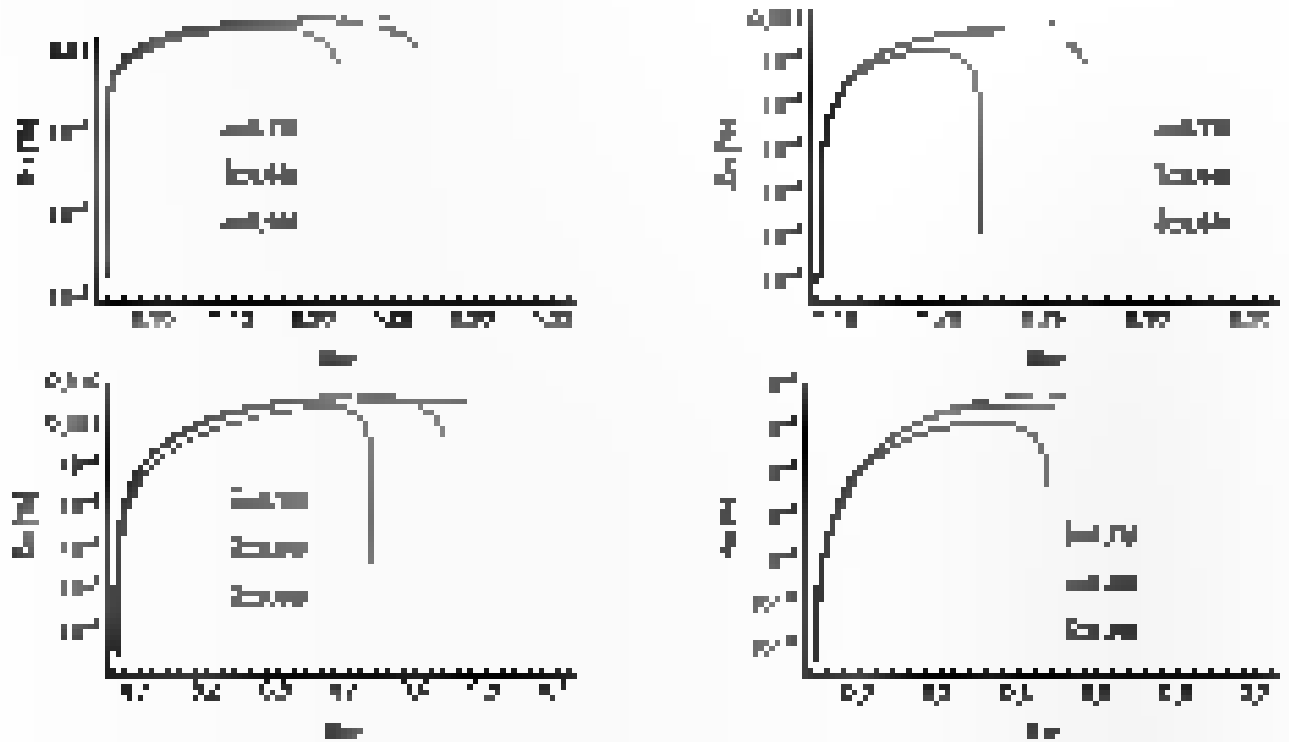


Fig. 11. - Variation of amplification factor Δ_{eff} with substrate concentration S_0 for $r=0.2$ (top left), $r=0.5$ (top right), $r=0$ (bottom left), and $r=0$ (bottom right).

Differentiating the black hole horizon radius r_h with respect to the following parameters for the total equations (4.64)

$$R_{\text{hor}} = \frac{r_h - r_{\text{Sch}}}{\sqrt{1 - \frac{2M}{r_h}}} = \frac{r_h - r_{\text{Sch}}}{r_h - r_{\text{Sch}}} = 1 \quad (4.65)$$

The above relations represent the physical boundary conditions that the horizon radius will be high (close to horizon) or much higher (close to spatial infinity) as the density ρ approaches the critical value ρ_c . With the new radial function

$$r = \frac{1}{3} R_{\text{hor}}, \quad (4.66)$$

the radial equation (4.64) becomes

$$\frac{d^2}{dr^2} u^2(r) + \left(\frac{1}{r} - \frac{r_{\text{Sch}}}{r^2} \right) u(r) = 0 \quad (4.67)$$

with

$$u = R_{\text{hor}} \left(1 - \frac{2M}{R_{\text{hor}}} \right)^{\frac{1}{2}} \frac{dr}{dR_{\text{hor}}} \quad (4.68)$$

where the Regge-Wheeler equation becomes a form of the Schrödinger equation ψ' is defined such that $\psi = -1/u$

$$\psi'' + \left(\mu^2(r) + \frac{1}{r} \right) \psi = 0, \quad \mu^2(r) = \frac{1}{r} - \frac{2M}{r^2} \quad (4.69)$$

To obtain a regularity condition at the horizon r_{Sch} we require that asymptotic behaviour of ψ is $\psi \sim \ln(r - r_{\text{Sch}}) + \text{const}$ [107]. This along with the limit of the asymptotic behaviour of ψ at spatial infinity (that takes $\psi \sim 1/r$) sets the regularity [108]

$$\frac{d}{dr} \psi = 0 \quad (4.70)$$

which the integrated effects of Ricci tensor, Weyl tensor, black hole and other horizon field may represent are represented as described by the known $\mu^2(r)$ in the horizon. The physical effect of the asymptotic horizon field is $\psi \sim \ln(r - r_{\text{Sch}}) + \text{const}$ and the asymptotic behaviour $\psi \sim 1/r$ at r_{Sch}

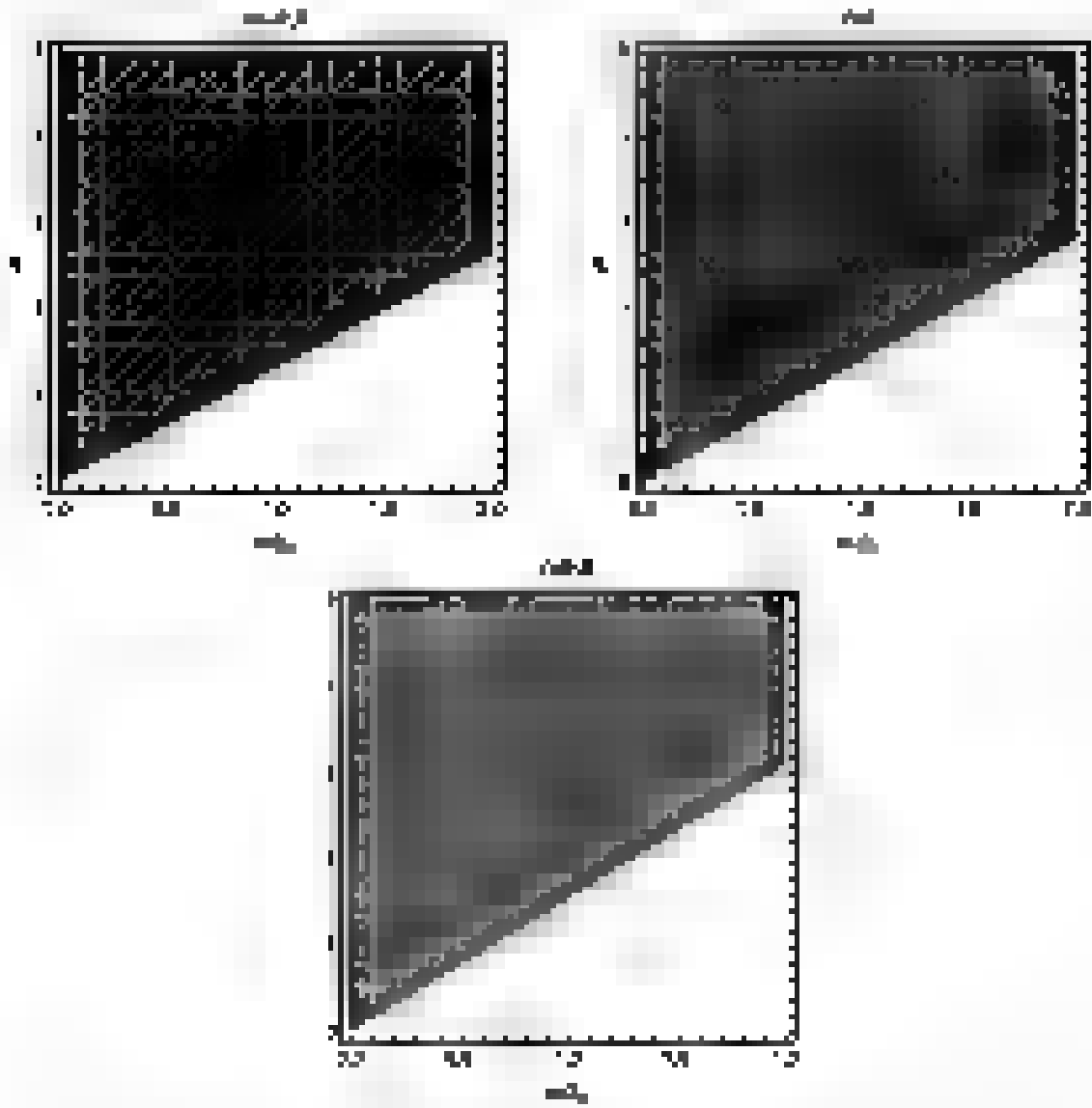


Figure 11: The function $f(x, y)$ for $\alpha = 0.1$ (top left), $\alpha = 0.2$ (top right), and $\alpha = 0.3$ (bottom center). The surfaces are smooth and show a slight increase in height as α increases.

Chapter 7

Constraining parameters involved in modified gravity using observational data for M_8^* black hole

The modified potential of gravity contains various free parameters. The recent data concerning M_8^* star light and stellar kinematics with the use of M81 project and red Wiltz ring galaxy ring kinematics as well as upcoming radio observations to constrain the free parameters involved in the gravitoid theories. With this purpose we carry up the shadow parameters, free parameters, constraining, in the gravitoid theories from the available because the fact that the fact hole is source and what is considered is played with the observed one for the M_8^* black hole. Comparison, we discuss the following uncertainties observed and estimated [10].

$$D = 12.4 \pm 0.4 \text{ kpc} \quad (11)$$

where D is the angular diameter M_8^* is the angular diameter. The boundary of the shadow is determined by the polar coordinate (R, ϕ) with the origin at the center of the shadow (ϕ_0, R_0 where $\phi_0 = \frac{2\pi}{3}$ and $R_0 = \frac{1}{\sqrt{2}}$).

If $\phi = 0$ and $\phi = 2\pi$ are the angular of the upper and lower boundary of the angular parameter $\phi = 0$ and $\phi = 2\pi$ be the angular boundary point $\phi = 0$ and $\phi = 2\pi$ over the angular R_{app} of the shadow given by [11]:

$$R_{\text{app}} = \frac{1}{\sqrt{2}} \int_0^{2\pi} d\phi \sqrt{A(\phi)} \quad (12)$$

where $A(\phi) = \frac{1}{2} \left(\frac{dR}{d\phi} \right)^2 = \frac{1}{2} \left(\frac{dR}{d\phi} \right)^2$ and $\frac{dR}{d\phi} = \frac{dR}{d\phi}$.

So the shadow beyond the distance from center of M_8^* is defined by [12]:

$$D = \frac{1}{\sqrt{2}} \int_0^{2\pi} d\phi \sqrt{A(\phi)} \quad (13)$$

We can consider the angular diameter of shadow, where D is defined,

$$D = \frac{1}{\sqrt{2}} \int_0^{2\pi} d\phi \sqrt{A(\phi)} \quad (14)$$

where $\frac{1}{\sqrt{2}} \int_0^{2\pi} d\phi \sqrt{A(\phi)}$ is the shadow size and $\frac{1}{\sqrt{2}} \int_0^{2\pi} d\phi \sqrt{A(\phi)}$ is the shadow size. In case M_8^* the mass has been estimated as $M = 10^{10} \pm 10^{11} M_{\odot}$ [13]. Using the mass (14), the shadow size D can be calculated for the M81 black hole. From the data available at the cosmological background from the observation of the shadow and the cosmological background (14) and the cosmological background.

5.1 Constraining the parameters ℓ for the Kerr-Schwarzschild black hole surrounded by plasma

Here, we will be considering the geometry using the spacings $P = 1$. Figures below correspond to different black hole values P_1 and ℓ in relation to the distance of the black hole, associated by parameter ℓ . The square of the distance $P = R^2$ and $1/P$ respectively.

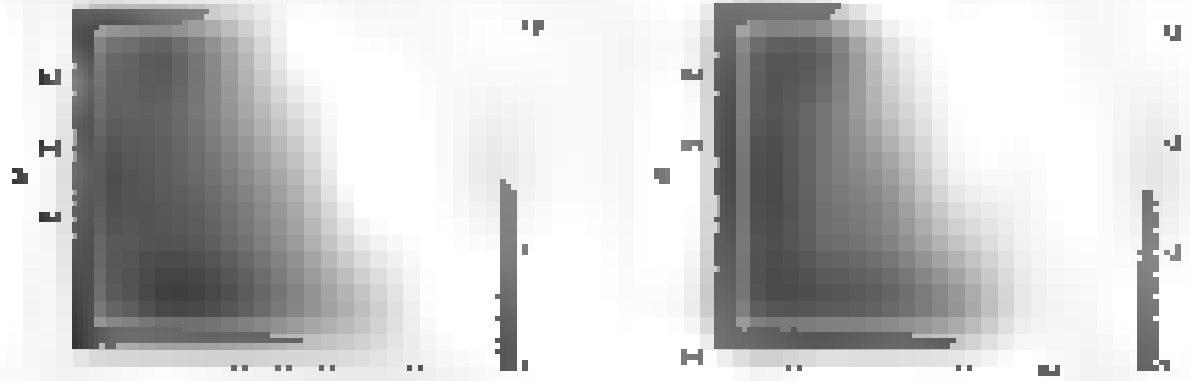


Figure 5. The distribution of black hole parameters P_1 and ℓ in relation to the distance of the black hole (not constant). The left one is for $P_1 = 0.001$ and $\ell = 0.001$. The right one is for $P_1 = 0.0001$ and $\ell = 0.001$. The angle of inclination is $\theta = 0$. The color bar is the logarithm of the density of the distribution.

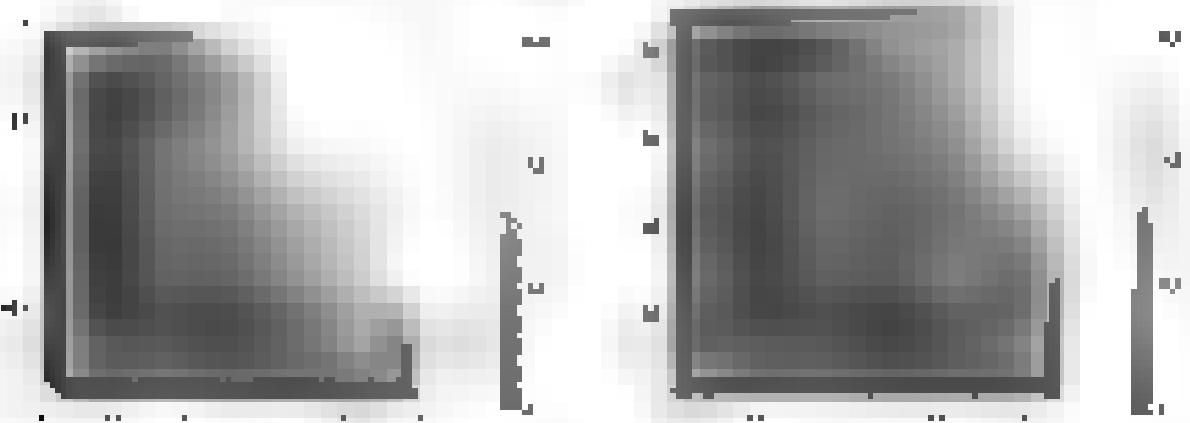


Figure 6. The distribution of black hole parameters P_1 and ℓ in relation to the distance of the black hole (not constant). The left one is for $P_1 = 0.001$ and $\ell = 0.001$. The right one is for $P_1 = 0.0001$ and $\ell = 0.001$. The angle of inclination is $\theta = 0$. The color bar is the logarithm of the density of the distribution.

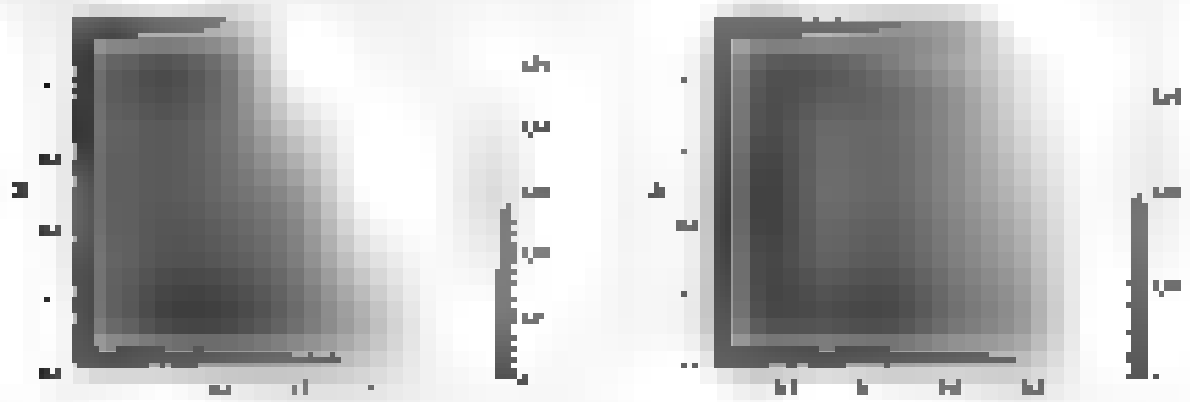


Figure 7. The distribution of black hole parameters P_1 and ℓ in relation to the distance of the black hole (not constant). The left one is for $P_1 = 0.001$ and $\ell = 0.001$. The right one is for $P_1 = 0.0001$ and $\ell = 0.001$. The angle of inclination is $\theta = 0$. The color bar is the logarithm of the density of the distribution.

3.1. CONSTRUCTION OF PARAMETER, FOR THE KANSI-GRA-LER BLACK HOLE
with Rotationally Symmetric

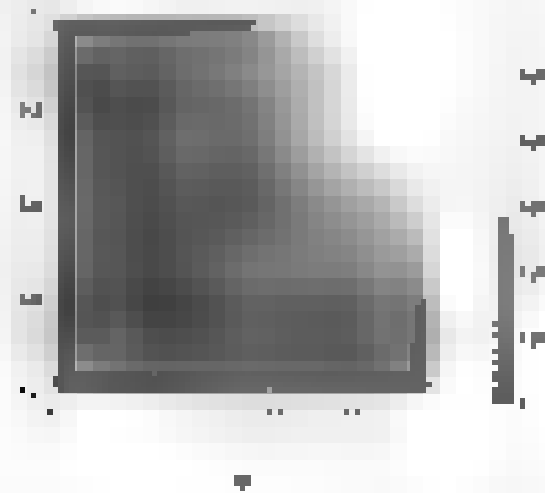


Figure 3.1 The above table for the density of black hole parameters. The density is $\rho = 1$ at $\theta = 0$ and $\phi = 0$ and $\rho = 0$ at $\theta = 2\pi$ and $\phi = 2\pi$.

Figure 3.2 The above table for the density of black hole parameters. The density is $\rho = 1$ at $\theta = 0$ and $\phi = 0$ and $\rho = 0$ at $\theta = 2\pi$ and $\phi = 2\pi$.

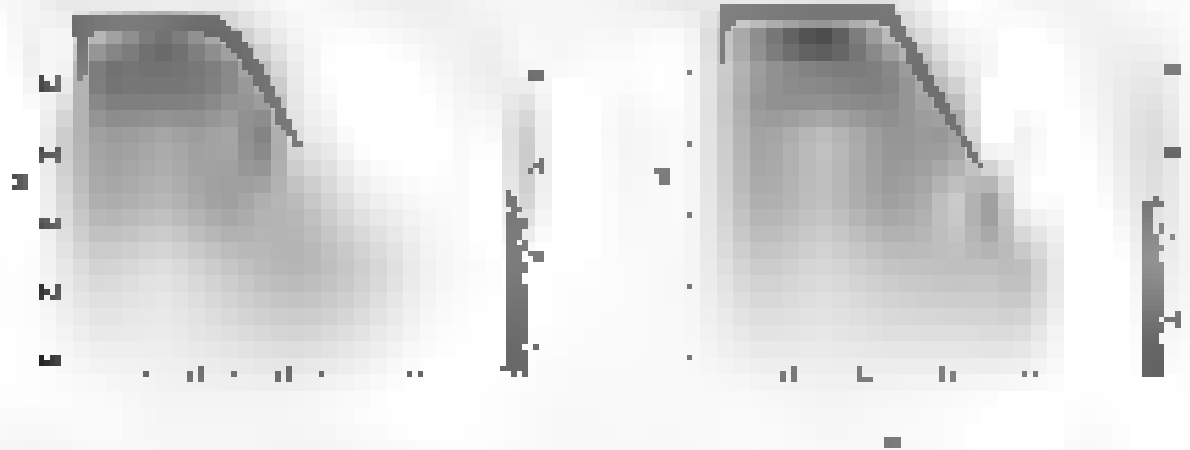


Figure 3.3 The above table for angular diameter θ for density of black hole parameters. The density is $\rho = 1$ at $\theta = 0$ and the angle of inclination is $\theta = 0$. The right axis is $\theta = 0$ and $\theta = 2\pi$. The left axis corresponds to $\theta = 0$ and $\theta = 2\pi$.



Figure 3.4 The above table for angular diameter θ for density of black hole parameters. The density is $\rho = 1$ at $\theta = 0$ and the angle of inclination is $\theta = 0$. The right axis is $\theta = 0$ and $\theta = 2\pi$, and the left axis corresponds to $\theta = 0$ and $\theta = 2\pi$.

Figure 1.10: The relationship between the number of black holes and the number of white holes in the universe.

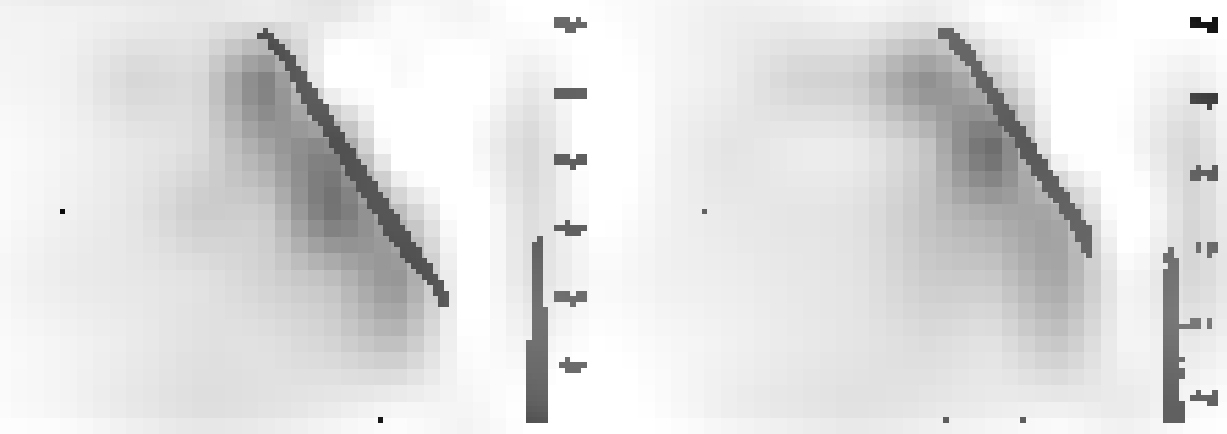


Figure 1.11: The relationship between the number of black holes and the number of white holes in the universe.

The relationship between the number of black holes and the number of white holes in the universe is a complex one. It is not clear whether the number of white holes is equal to the number of black holes, or if there are more white holes than black holes. This is a topic that is still being researched.

N _{BH}	N _{WH}	N _{BH} / N _{WH}	N _{BH}	N _{WH}	N _{BH} / N _{WH}
0	0	0/0	0	0	0/0
1	1	1/1	1	1	1/1
2	2	2/2	2	2	2/2
3	3	3/3	3	3	3/3
4	4	4/4	4	4	4/4
5	5	5/5	5	5	5/5
6	6	6/6	6	6	6/6
7	7	7/7	7	7	7/7
8	8	8/8	8	8	8/8
9	9	9/9	9	9	9/9
10	10	10/10	10	10	10/10
11	11	11/11	11	11	11/11
12	12	12/12	12	12	12/12
13	13	13/13	13	13	13/13
14	14	14/14	14	14	14/14
15	15	15/15	15	15	15/15
16	16	16/16	16	16	16/16
17	17	17/17	17	17	17/17
18	18	18/18	18	18	18/18
19	19	19/19	19	19	19/19
20	20	20/20	20	20	20/20

The relationship between the number of black holes and the number of white holes in the universe is a complex one. It is not clear whether the number of white holes is equal to the number of black holes, or if there are more white holes than black holes. This is a topic that is still being researched.

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31. CONSTRUCTION OF A BOUNDARY FOR THE SPACE OF A LOG-BLACK HOLE
(attributed to the author)

We consider a constant M "background" $(\rho, \theta, \phi, \tau)$ with $\rho = 0, \theta = 0, \phi = 0, \tau = 0$. We also consider the set $\mathcal{S} = \{(\rho, \theta, \phi, \tau) \mid \rho = 0, \theta = 0, \phi = 0, \tau = 0\}$ and the knowledge that $\mathcal{S} = \{(\rho, \theta, \phi, \tau) \mid \rho = 0, \theta = 0, \phi = 0, \tau = 0\}$. Here we find an upper bound of \mathcal{S} which means that $\mathcal{S} \subseteq \mathcal{S}$.

3E. CONTINUING THE CHANGE PARAMETERS IN REPRESENTING BLACK HOLE IN WOLFF-CHAIKIN CONSTRUCTION WITH $\lambda = 0.01$

7.1 Continuing the change parameter b in Kerr-Schild-like Black hole in vacuum from the observed data for $b(3)$

THEY are shown the Kerr-Schild-like black hole in vacuum part of the black hole in the metric in which the metric is continued to vacuum the change parameter b using the construction [1] and the data are given in Eq. (7.1) and Eq. (7.1). In the figure below, the Kerr-Schild-like black hole in vacuum for Kerr-Schild-like black hole in vacuum is shown for $b = 10$ and $b = 1$ respectively.

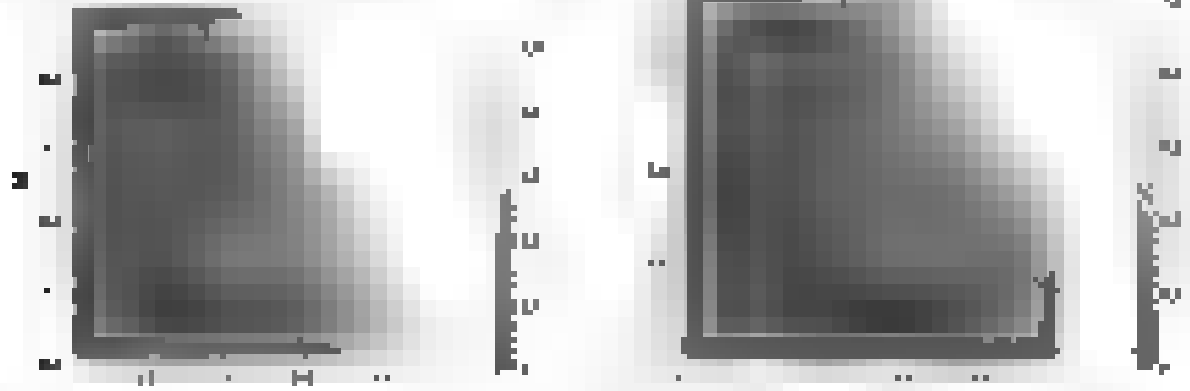


Figure 1d: The left is $b = 10$ and the right is $b = 1$. b is the change parameter, $b(3) = 3$. The black hole is represented by $b = 3$.

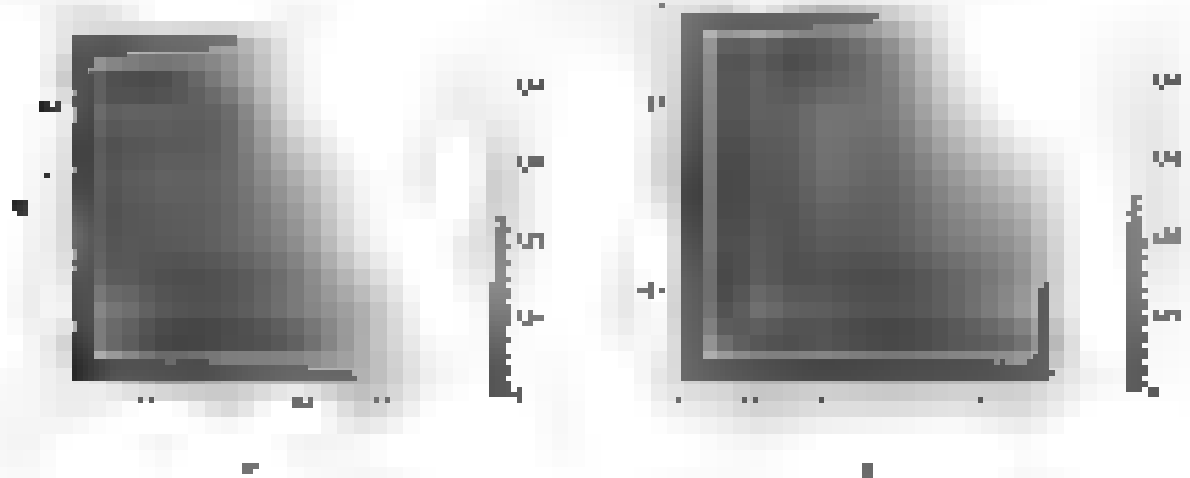


Figure 1e: The left is $b = 10$ and the right is $b = 1$. b is the change parameter, $b(3) = 3$.

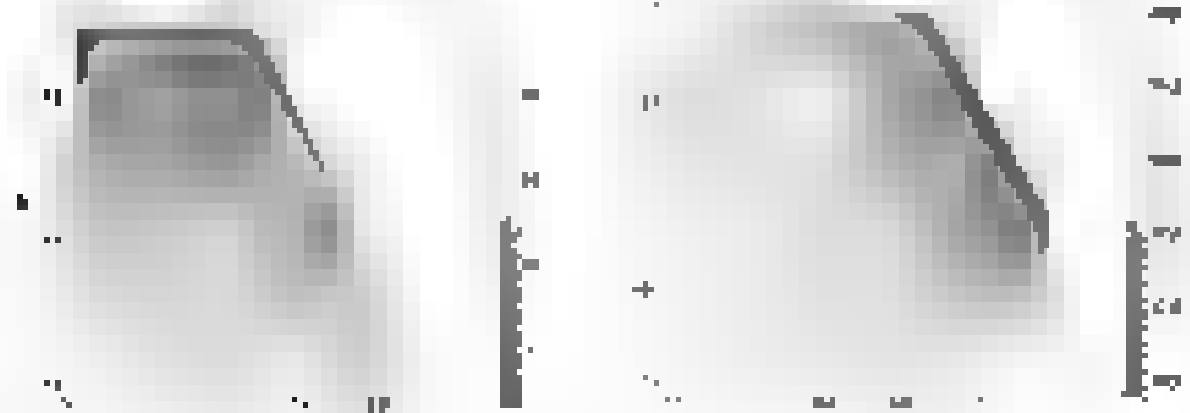


Figure 1f: The left is $b = 10$ and the right is $b = 1$. b is the change parameter, $b(3) = 3$. The black hole is represented by $b = 3$.

5. Constraining the non-conductivity parameter λ from the observed data for $\beta = 1$

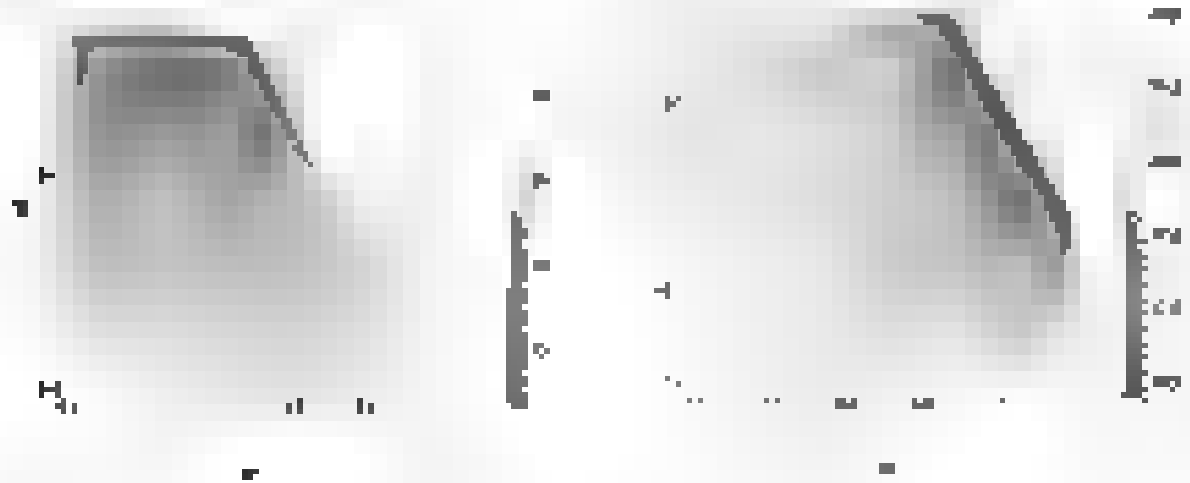


Figure 7: The left panel (a) shows the relationship between H_0 , H_1 and the non-conductivity parameter λ for the distribution angle is 30° . The right panel (b) corresponds to 15° case.

From Figure 7 (a) and (b) we can clearly observe that the relationship between H_0 and H_1 is affected by the distribution angle. When the distribution angle is 30° and for the distribution angle is 15° , the non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively. Figure 8 (a) and (b) show the relationship between H_0 and H_1 for the distribution angle is 30° and 15° respectively. The non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively. The non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively. The non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively.

5.3 Constraining the non-conductivity parameter λ from the observed data for $\beta = 1$

This section discusses constraining the parameter λ . We first show the results in Figure 7 (a) and (b) for the distribution angle is 30° and 15° respectively. In the figure, the surface shows the relationship between H_0 and H_1 for the distribution angle is 30° and 15° respectively. The non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively.

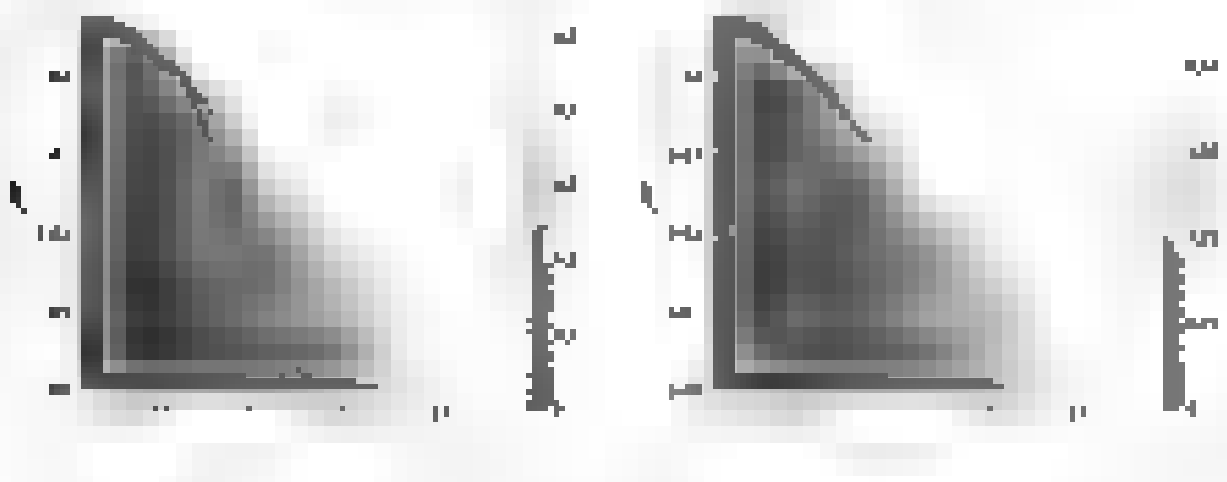


Figure 8: The left panel (a) shows the relationship between H_0 , H_1 and the non-conductivity parameter λ for the distribution angle is 30° . The right panel (b) corresponds to 15° case.

From the plot (a) and (b), we can clearly observe that the relationship between H_0 and H_1 is affected by the distribution angle. When the distribution angle is 30° and for the distribution angle is 15° , the non-conductivity parameter λ is constrained by the observed data for $\beta = 1$ and $\beta = 0.5$ respectively.

5. CONSTRAINING THE NON-COARCTATE PARAMETER λ FROM THE OBSERVED DATA FOR 10°

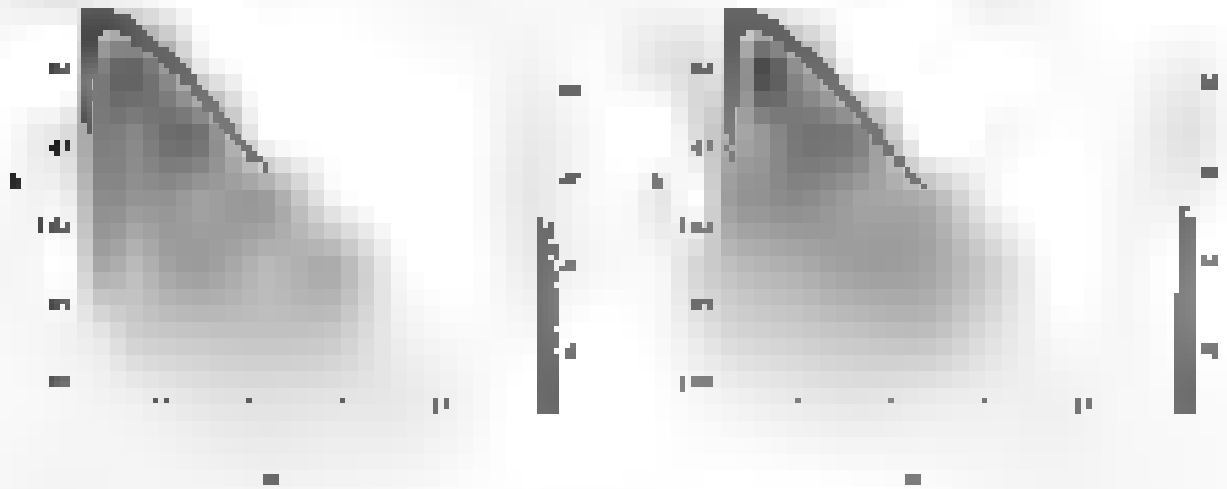


Figure 14: For the left panel the observation angle is 10° and for the right panel the observation angle is 17° . The NHT model is represented by $\lambda = 10.0$.

From the plot (Figure 14) we can conclude that for observation angle $\theta = 10^\circ$ and $\theta = 17^\circ$ the variable $\lambda_2 = 10$ is λ_2 which is highly standard for these parameter space. The circular symmetry in the λ_2 direction \Rightarrow no deformation term. The variable λ_2 which is the standard map to the mean direction of the variable [12], is defined by [17]

$$\lambda_2 = \frac{\Delta F}{\Delta X} = \frac{\Delta}{\Delta x} \quad (24)$$

We should have $\lambda_2 = 0$ in accordance with the λ_2 observation ≈ 0.07 [14]. Also, they \Rightarrow a smaller Δx if dividing ΔF about two of Δ indeed corresponds to a $\Delta F = 10$ in the figure below and two λ_2 is shown for non-coarctate Kant-like black hole for inclined angle $\theta = 10^\circ$ and $\theta = 17^\circ$ respectively.

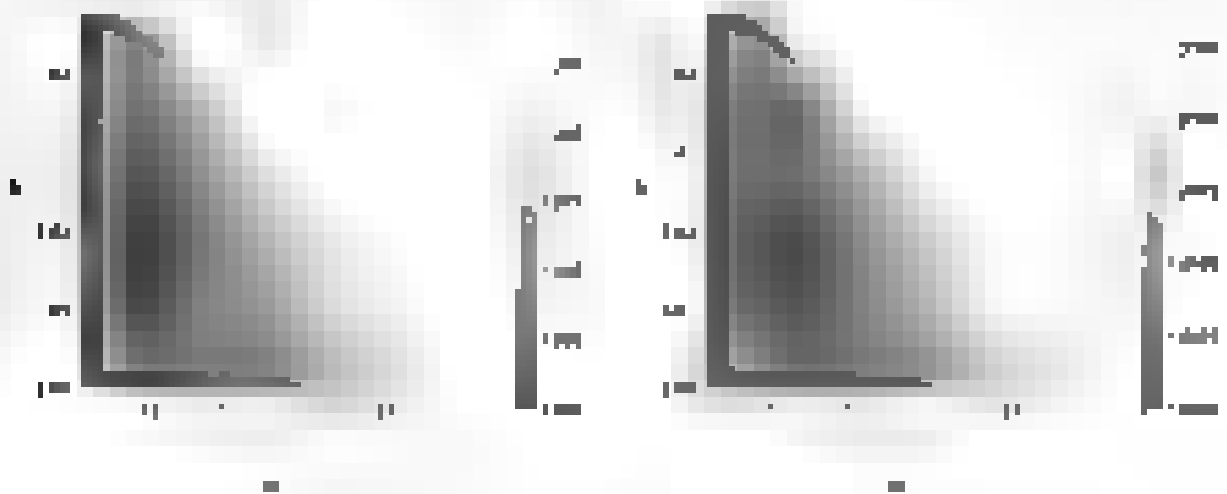


Figure 15: For the left panel the observation angle is 10° and for the right panel the observation angle is 17° .

From the plot above we can see that the variable $\lambda_2 = 10$ is standard for the value parameter space of non-coarctate Kant-like black hole. Thus, non-coarctate Kant-like black hole are remarkably consistent with NHT angular of 40° . Therefore, we cannot rule out non-coarctate Kant-like black hole from the observational data of 40° black hole shadow.

We can take the limit of the parameter λ_2 associated with the non-coarctate λ_2 at the 1 year from $\lambda_2 = 10$ as we can distinguish the limit of the parameter λ_2 in [17] by modeling 90° black hole as Kant-like hole, the author of the paper [17] obtained a lower limit of λ_2 by 40°

5 CONSTRAINING THE NON-CLUSTERING PARAMETER α FROM THE OBSERVED DATA POINTS

Next, we are keeping the null under consideration, $\mathcal{H}_0(\mathbf{P})$ or put the explicit statement for α as $\mathcal{H}_0(\mathbf{P}, \alpha) : \alpha \leq 0$ using the experimental results in \mathcal{D}_1 (\mathcal{D}_2) and $\mathcal{D}_3 : \alpha \leq 0$ as well as the rejection region $\mathcal{R} = \{P \in \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3 \mid \text{we do reject } \mathcal{H}_0(\mathbf{P}, \alpha) : \alpha \leq 0\}$. In order to make integrable under the bounds $\alpha \in [\beta, \infty)$ ($\alpha \in (-\infty, \beta]$) and the exponential convergence $\Delta \alpha^2$ and $\delta : (0, \infty) \rightarrow \mathbb{R}$ we get bound on the parameter α which is linked with the non-clustering hypothesis. We find that the parameter $\alpha \in [\beta, \infty)$ and $\Delta \alpha^2 \in \mathbb{R}^+$ is integrating to take an upper bound of δ_α which is equal to be 3.0000000000. To the best of our knowledge, the bound of the parameter α from the studies of the observational black hole binary system has not reported so far.

Chapter 3

Learning Objectives: After reading this chapter, you should be able to:

- 1. Explain the importance of the background of a project.
- 2. Identify the key stakeholders in a project.
- 3. Describe the role of the project manager.
- 4. Explain the importance of communication in a project.
- 5. Identify the key elements of a project plan.
- 6. Describe the role of the project sponsor.
- 7. Explain the importance of risk management in a project.
- 8. Identify the key elements of a risk management plan.
- 9. Describe the role of the project steering committee.
- 10. Explain the importance of quality management in a project.
- 11. Identify the key elements of a quality management plan.
- 12. Describe the role of the project manager in quality management.
- 13. Explain the importance of time management in a project.
- 14. Identify the key elements of a time management plan.
- 15. Describe the role of the project manager in time management.
- 16. Explain the importance of cost management in a project.
- 17. Identify the key elements of a cost management plan.
- 18. Describe the role of the project manager in cost management.
- 19. Explain the importance of procurement management in a project.
- 20. Identify the key elements of a procurement management plan.
- 21. Describe the role of the project manager in procurement management.
- 22. Explain the importance of human resource management in a project.
- 23. Identify the key elements of a human resource management plan.
- 24. Describe the role of the project manager in human resource management.
- 25. Explain the importance of communication management in a project.
- 26. Identify the key elements of a communication management plan.
- 27. Describe the role of the project manager in communication management.
- 28. Explain the importance of stakeholder management in a project.
- 29. Identify the key elements of a stakeholder management plan.
- 30. Describe the role of the project manager in stakeholder management.

1. Explain the importance of the background of a project.

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21. Describe the role of the project manager in procurement management.

22. Explain the importance of human resource management in a project.

23. Identify the key elements of a human resource management plan.

24. Describe the role of the project manager in human resource management.

25. Explain the importance of communication management in a project.

26. Identify the key elements of a communication management plan.

27. Describe the role of the project manager in communication management.

28. Explain the importance of stakeholder management in a project.

29. Identify the key elements of a stakeholder management plan.

30. Describe the role of the project manager in stakeholder management.

A. STRONG AND HYPERGRAVITATIONAL COLLING

where α is the affine parameter along the geodesics. From Eq. (18) and Eq. (18.7) we obtain the following equation for the null geodesics

$$\frac{dr}{ds} = -\beta = -\beta \frac{d^2}{ds^2} \frac{1}{r^2} \quad (18.9)$$

Thus, the effective potential $V_{\text{eff}}(r)$ comes out to be

$$V_{\text{eff}}(r) = \frac{d^2 V_{\text{eff}}}{ds^2} = \frac{d^2}{ds^2} \left(\frac{1}{r^2} + \alpha \frac{2\beta(1-\beta)}{r^3} \right) \quad (18.10)$$

For circular photon orbits of red $\beta = 1$, the effective potential takes exactly the form as

$$\frac{dV_{\text{eff}}}{ds} = 0 \quad (18.11)$$

which yields

$$\frac{F(r)}{r^3} = \frac{2}{3r^2} \quad (18.12)$$

With the use of the above equation we have

$$(2r - 3)(1 - \beta_2) - \beta_2 r^2 = 0 \Rightarrow (1 - \beta_2) + \frac{1}{r} = 0 \quad (18.13)$$

Now, when $r = 0$, the condition $\frac{d^2 V_{\text{eff}}}{ds^2} < 0$ is maintained which indicates that the orbits of the photons to generate circular photon orbits are unstable under perturbations. The photons from the infinitely large distant matter that approached the black hole with initial distance r_0 from the black hole to maximum distance r_2 will be deflected back, asymptotically to infinity. The impact parameter b_0 and the maximum distance r_2 are related to each other through Eq. (18.13) as

$$b_0 = \frac{1}{r_0} \frac{\sqrt{1 - \beta_2}}{\sqrt{1 - \beta_2}} \quad (18.14)$$

where β_2 can be found as

$$V_{\text{eff}}(r_2) = \frac{1}{r_2^2} + \alpha \frac{1}{r_2} = \frac{1}{r_2} \quad (18.15)$$

The impact parameter corresponds to $r_2 = 0$.



Figure 1: The deflection angle α and the impact parameter b_0 as functions of r_2 .

We can infer from Fig. 1 that the radius r_2 and the impact parameter b_0 as $r_2 \rightarrow 0$ decreases with the increase in α_0 , and that the deflection angle α increases with the increase in r_2 . Let us look into the gravitational deflection angle of light. It has been demonstrated in the article [24, 25] as follows

$$\alpha = \alpha_{\text{GR}} + \alpha_{\text{NG}} \quad (18.16)$$

where

$$\gamma(r) = \frac{r^2}{b_0} \left(\frac{1}{2\sqrt{a(r)c(r)}} \frac{1}{\sqrt{1 - \frac{2M(r)}{r}}} - 1 \right) \quad (17)$$

In the absence of a black hole, the charge is due to a photon whose trajectory has impact parameter b_0 and hence, from Eq. (18), we find that the deflection angle is zero. According to relation (16), we can conclude that as deflection angle α is small, we have small distances in the spatial parameter b_0 and accordingly, a small period of exposure τ which implies the completion of exposure within the path of the photon encompassing the black hole. Also, this special situation is reported in our website <http://www.astro.umd.edu/~graystonski/physics/physics.html> where the curvature distance approaches the radius of the photon sphere. In such situation the deflection angle changes. However, for $b_0 = b_{ph}$, photon is not captured by the black hole. In this strong field limit, however, nothing stops photon close to the black hole.

Employing an asymptotic approximation method used in [14], we expand the deflection angle about the photon sphere when radius is diverging from which nonzero deflection is a non-trivial task. $\gamma(r)$ is a function of r and, the integral (17) can be written down as [15]

$$\gamma(r_0) = \int_{r_0}^{\infty} R(x) dx, \quad (18)$$

where $R(x)$, x_0 and $\gamma(r_0)$ are given by

$$x_0 = r_0 = \frac{2\sqrt{M(r_0)}}{1 + \sqrt{1 - 2M(r_0)/r_0}} \quad (19)$$

$$R(x) = \frac{1}{\sqrt{2M(x) - \frac{r_0^2}{4M(x)} - \frac{r_0^2}{4M(x)^2}}} \quad (20)$$

The function $R(x)$ is regular for all possible choices of r_0 and x_0 because the function $\gamma(r_0)$ is not divergent when $r_0 \rightarrow 0$ or $r_0 \rightarrow \infty$ and the divergence of $R(x)$ is $\sim 1/\sqrt{x}$ as $x \rightarrow 0$ and the function is bounded within the square root of the Eq. (20) as $x \rightarrow \infty$ since and also the required limits of appropriate approximation is given as the divergence of $R(x)$ is the same as $x \rightarrow 0$.

$$R(x) = \frac{1}{\sqrt{2M(x) - \frac{r_0^2}{4M(x)} - \frac{r_0^2}{4M(x)^2}}} \quad (21)$$

where $M(r_0)$ and $m(r_0)$ are using the following algebraic expression

$$m(r_0) = \frac{b_0}{4r_0^2} (1 + \sqrt{1 - 2M(r_0)/r_0} - M(r_0)/r_0) \quad (22)$$

$$M(r_0) = \frac{(2m(r_0) - \sqrt{2M(r_0)/r_0})^2 m(r_0) + r_0^2}{4r_0^2} = \frac{b_0}{4r_0^2} (M(r_0)/r_0 - M(r_0)^2/r_0) \quad (23)$$

The integral is split into integral I & II over the following two integrals [14]

$$\gamma(r_0) = \gamma_1(r_0) + \gamma_2(r_0) \quad (24)$$

Explicitly, the integrals are

$$\gamma_1(r_0) = \int_{r_0}^{\infty} R(x) dx, \quad (25)$$

$$\gamma_2(r_0) = \int_{r_0}^{\infty} R(x) dx, \quad (26)$$

Here γ_1 is $\gamma_1(r_0) = \gamma_1(r_0)$ and γ_2 is $\gamma_2(r_0) = \gamma_2(r_0)$. Therefore, $\gamma_1(r_0) = \gamma_1(r_0)$ which makes the integral (24) divergent. $\gamma_2(r_0) = \gamma_2(r_0)$ is bounded. $\gamma_2(r_0)$ has an asymptotic behavior as $r_0 \rightarrow 0$ as

$$\gamma_2(r_0) = \gamma_2(r_0) = \gamma_2(r_0) \frac{1}{\sqrt{1 - 2M(r_0)/r_0}} \log \frac{1 + \sqrt{1 - 2M(r_0)/r_0} + \sqrt{1 - 2M(r_0)/r_0}}{1 - \sqrt{1 - 2M(r_0)/r_0}} \quad (27)$$

We also expand $\gamma_1(x_2)$ in the following fashion

$$\gamma_1(x) = \frac{m(r_0, r_m)}{M(r_m)} \sqrt{p - x} + C_1(x_1 - x_m) \quad (3.27)$$

and make the change up to $C_1(r_m - x_m)$ in (3.26), we may use the integral $P(x)$ as

$$C_1(x_1 - x_m) = \log \frac{1}{r_1} + F_1(x_1(r_m - x_m)) \quad (3.28)$$

where \log and $F_1(x)$ given by

$$\frac{M(r_m)}{\sqrt{p - x_m + 1}} \quad (3.29)$$

$$F_1 = \frac{\lambda(r_1)}{\sqrt{p(r_1)}} \log \frac{M(r_m)}{r_1} \quad (3.30)$$

if we take equal $F_1(x)$ to $F_1(x)$ at $x_1 = x_m$ and return back up to $\gamma_1(x_1 - x_m)$ with that we will have

$$\gamma_1(x_1) = \frac{m(r_1, x_m)}{M(r_m)} + F_1(x_1(r_m - x_m)) \quad (3.31)$$

$$= F_1(x_1(r_m - x_m)) \quad (3.32)$$

where

$$F_1 = F_1(x_1(r_m)) \quad (3.33)$$

Therefore we must expand the $\gamma_1(x)$ about r_m where γ_1 is 0

$$r_m - r_m = F_1(x_1 - x_m) \quad (3.34)$$

where

$$F_1 = \frac{1}{2} \frac{m(r_m)}{r_m^2} + \frac{M(r_m)}{r_1 - 1} \quad (3.35)$$

With the help of equations (3.34) and (3.35) we obviously have an expression of the angle of deflection in terms of the impact parameter

$$\alpha = \theta_1 - \theta_2 = \frac{b_1}{r_m} + F_1(x_1 - r_m) \quad (3.36)$$

$$\text{where} \quad (3.37)$$

$$F_1 = \frac{1}{r_1} \frac{M(r_1)}{r_1 - 1} \quad (3.38)$$

$$F_1 = F_1 + \log \frac{M(r_1)}{r_1 - 1} \quad (3.39)$$

And this result is the same as the solution of strong gravity solutions and weak gravity. Therefore, $\gamma_1(x) = \gamma_1(x) + F_1(x)$ and $\gamma_1(x) = \gamma_1(x) + F_1(x)$ is the same as the result of

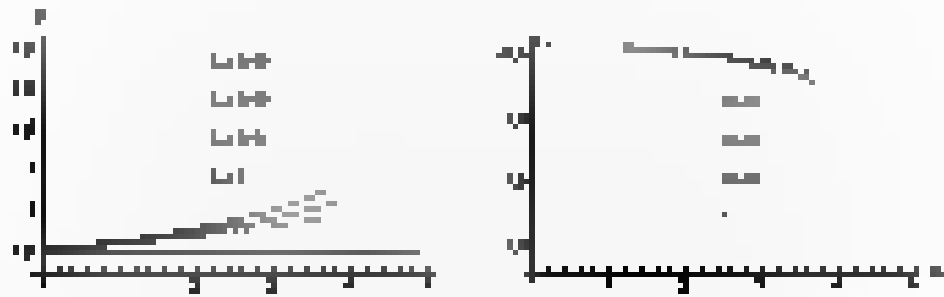


FIGURE 13. Plot showing the variation of the surface area of the apparent horizon and the surface area of the apparent horizon.

As seen in Figure 13, and for Table 3, it is evident that the surface area of the apparent horizon decreases with the increase of the dimensionless parameter x . The value of x is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter. The plots are similar to the plots for $L=1$. The value of L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter. In the following (Fig. 14), the variation of the surface area of the apparent horizon with respect to the dimensionless parameter x is presented graphically.

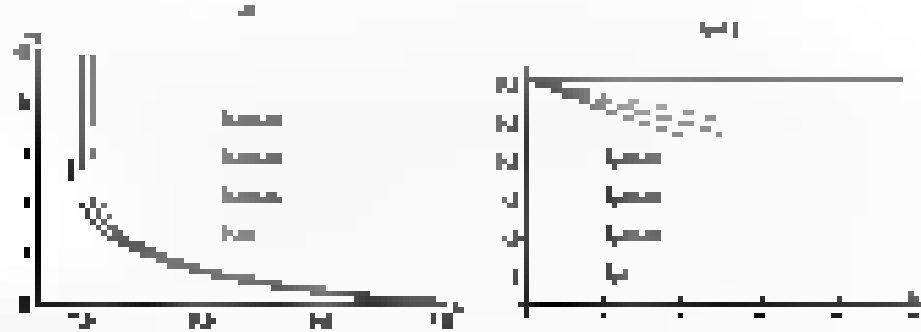


FIGURE 14. The variation of the surface area of the apparent horizon with respect to the dimensionless parameter x for different values of L with $L=1$. The right plot shows the variation of the surface area of the apparent horizon with respect to x with $L=1$.

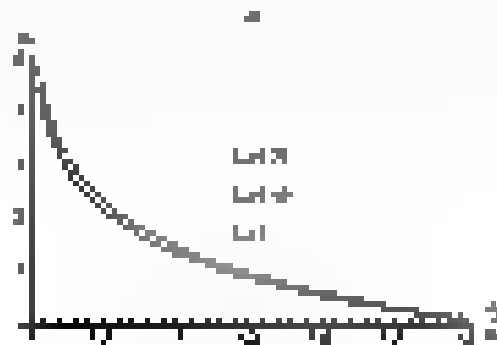


FIGURE 15. Plot showing the variation of the surface area of the apparent horizon with respect to the dimensionless parameter x with $L=1$.

From Figure 15, we see that the surface area of the apparent horizon decreases with the increase of the dimensionless parameter x . The value of x is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter. The plots are similar to the plots for $L=1$. The value of L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter. In the following (Fig. 16), the variation of the surface area of the apparent horizon with respect to the dimensionless parameter x is presented graphically. Here $L=1$ and $L=1000$ for different values of x . The dimensionless parameter x is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter, L is the dimensionless parameter.

m	k	Δ	$t_{\text{in}} \Delta$	H	IF	$d_{\text{in}} \Delta$
1	0.40	0.0441	0.000001	25.0000	0.00000001	0.00000001
	0.50	0.0100	0.000000	10.0000	0.000000	0.000000
	0.60	0.0077	0.000000	2.0000	0.00000001	0.00000000
2	0.40	0.0000	0.000000	0.0000	0.00000000	0.00000000
	0.50	0.0000	0.000000	0.0000	0.00000000	0.00000000
	0.60	0.0000	0.000000	0.00000000	0.00000000	0.00000000
3	0.40	0.0000	0.000000	0.00000000	0.00000000	0.00000000
	0.50	0.0000	0.000000	0.00000000	0.00000000	0.00000000
	0.60	0.0000	0.000000	0.00000000	0.00000000	0.00000000

Table 1: The binary results for the test of the statistical black hole and detection of them in the test from different sample black hole ($\Delta_{\text{in}} = 0$). Here, Δ is the difference between the previous value and the value.

the angular separation of the image from the source θ can be written as

$$\theta = \theta' - \Delta \log \frac{R D_{\text{in}}}{r_s} = \theta' - \Delta \mu_1 \approx \theta' \quad (6.9)$$

Here, θ' is the distance between the observer and the black hole. If we draw a line between the center of the black hole and an observed red shift of the lens between the observer and the point of the source distribution, the angle of detection is defined as $\theta' = \theta_{\text{in}} = \theta$.

6.2 Observables in the strong field limit

In the strong field limit, we can use the equation for the position and separation of multiple images with the help of Eq. (6.2) and the equation derived in the previous section. The separation between the two images is the distance between the two images in the plane of the image separation, between the source and the lens q given by

$$q = d - \frac{r_s}{D_{\text{in}}} \Delta \approx \quad (6.10)$$

where Δ is the lens separation, $D_{\text{in}} = D_{\text{in}}/D_{\text{in}}$ is the distance between the observer and the source and $d = \mu_1 \log \frac{R D_{\text{in}}}{r_s}$ is the difference $\mu_1 \log \frac{R D_{\text{in}}}{r_s}$. By the geometry of the image separation, the bending number Δ is the angle of the black hole in the plane of the image separation.

We can now use the previous equation to calculate the angle θ . We therefore need to find the angle of the black hole in the plane of the image separation.

$$\theta = \theta' - \Delta \quad (6.11)$$

The value θ is given by

$$\theta = \frac{b}{r_s} + \quad (6.12)$$

$$r_0 = \frac{L_0^2}{4\pi\epsilon_0} \quad (3.33)$$

Therefore, we consider $\delta\mathbf{r} = \mathbf{r} - \mathbf{r}_0^0$ and expand $\alpha_0(\mathbf{r})$ around $\mathbf{r} = \mathbf{r}_0^0$ in order to compute offset angle. Unusually, we deal with the following differential equation of the offset angle

$$\Delta\alpha_0 = \frac{\partial\alpha_0}{\partial r_0} \Delta r_0 \quad (3.34)$$

Using the expansion $r = r_0^0 + \Delta r_0$ in Eq. (3.33) we obtain the first algebraic expansion of r_0

$$r_0 = r_0^0 + \Delta r_0 = \frac{4\pi\epsilon_0 L_0^2}{F_{\text{ext}} - \frac{D_L}{D_{\text{off}}}} \Delta r_0 \quad (3.35)$$

Since the condition $\frac{D_L}{D_{\text{off}}} \ll 1$ holds for physically reasonable cases, we can neglect the second term in the above equation as compared to the last term. Hence, the position of the i^{th} image is separated by $L_0(1 + \epsilon)$

$$r_0 = r_0^0 + \frac{r_0^0 \epsilon}{\frac{D_L}{D_{\text{off}}}} \quad (3.36)$$

When $\epsilon = \frac{D_L}{D_{\text{off}}}$ the position of the image and the source coincide. It implies that the i^{th} source has a ring around any correction. Therefore, Eq. (3.36) will describe an image on the same side of the radiation emitting. To get the position of the image on the other side of the source, we need to replace ϵ with $-\epsilon$.

The study of the magnification of the image is also interesting. Magnification of the image is another good source of information [64], in addition to the position of the source. The magnification of the image is given by

$$\mu_0 = \frac{1}{2} \frac{\Delta_0}{\Delta r_0} \quad (3.37)$$

$$= \mu_0 \frac{d_0}{2L_0 + \frac{1}{2} \frac{D_L}{D_{\text{off}}}} \quad (3.38)$$

It is transparent from the Eq. (3.38) that the magnification decreases with an increase in L_0 . It is beneficial to consider special situation where the observed single ring image $r_0 = r_0^0$ is surrounded by other (multiple) ring image (ring image sets). The other (multiple) images are separated to be rubbed together at r_0 . These combinations of an increasing or decreasing are given by

$$r_0 = \frac{d_0}{\Delta r_0} \quad (3.39)$$

$$\epsilon = \pm \mu_0 \frac{d_0}{L_0} \quad (3.40)$$

$$\frac{1}{\sum_{i=1}^N (1 + \epsilon_i)} = \frac{1}{N} \quad \text{or} \quad (1 + \epsilon) = \frac{F_0}{L_0 D_{\text{off}}} \quad (3.41)$$

Here d_0 gives the position of the observed partial image, μ_0 gives the relative separation between the first image and the observed image and ϵ gives the ratio of the flux from the first image to flux from the remaining images. It can be noted that although L_0 and d_0 depend on the distance between black hole and the observed image, they do not depend on this.

Now we consider the supermassive black holes $\sim 10^6 M_\odot$ or more in double ring image separation (ring image pair) by using the eqn. (3.41). We consider the $\sim 10^6 M_\odot$ black holes are $\sim 10^{-10} M_\odot$ and $\sim 10^{-11} M_\odot$ respectively and the distance of them are $\sim 10^3$ AU or $\sim 10^4$ AU and $\sim 10^5$ AU respectively [64, 65].

Fig. 3 shows that the distance of binary black holes depends on L_0 for different values of F_0 in the L_0 - F_0 and L_0 - D_{off} plane. We can also see the position r_0 and ϵ (ring distance) with the variation of L_0 , whereas they are two curves with L_0 increasing. The ring separation

A. AVERAGE INSTANTANEOUS FIELD FLUX

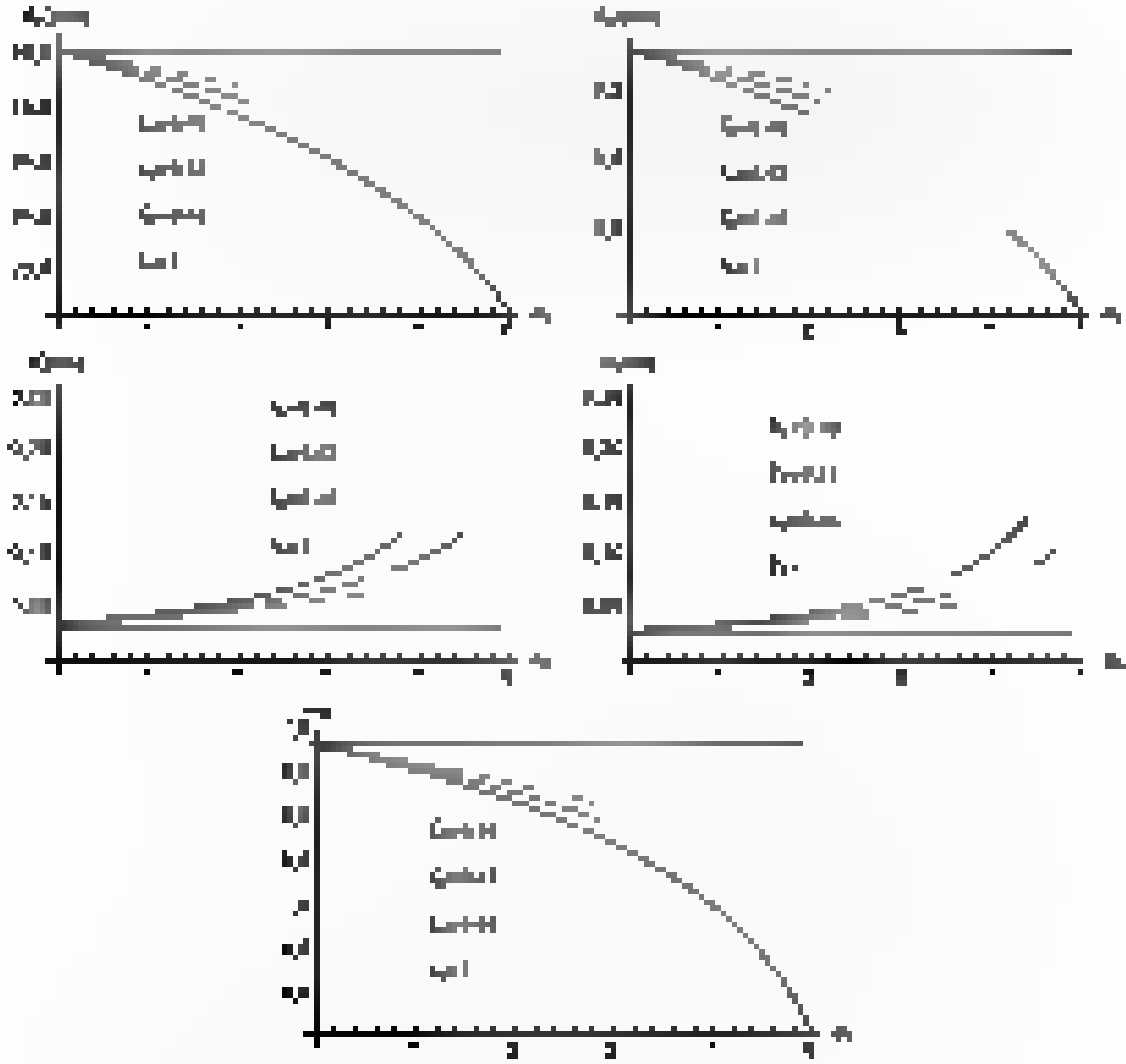


Figure 4: Average instantaneous field flux $\langle B_z \rangle$ as a function of η for $\alpha = 0.1$ (left panel) and $\alpha = 0.2$ (right panel) at $t = 0.5$.

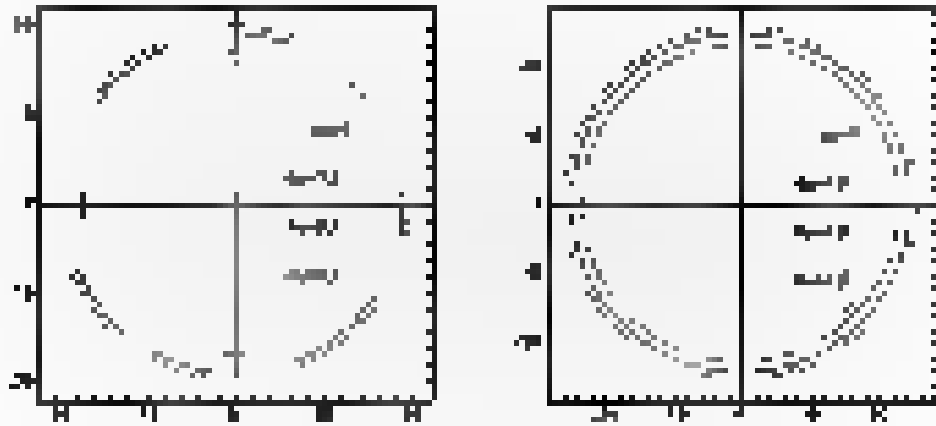


Figure 5: Average instantaneous field flux $\langle B_z \rangle$ as a function of η for $\alpha = 0.1$ (left panel) and $\alpha = 0.2$ (right panel) at $t = 0.5$.

m_1	Sgr A'		M31		m_2
	$d_{\text{BH}} [\text{km}]$	$\langle \dot{\gamma}_{\text{BH}} \rangle$	$d_{\text{BH}} [\text{km}]$	$\langle \dot{\gamma}_{\text{BH}} \rangle$	
0.40	15.2524	8.7404067	18.5951	8.0214593	4.51479
	17.1014	8.7140000	18.1349	8.307791	4.87058
	19.5079	8.69124	17.7939	8.554677	5.23079
0.43	14.4250	8.7402919	19.9117	8.020	4.77718
	15.0254	8.7407440	19.6217	8.313403	4.90777
	15.9614	8.7404497	19.7441	8.514443	5.11546
0.46	14.1347	8.770114	19.3660	8.364443	5.10211
	15.1173	8.7401109	18.6224	8.366467	4.91843
	17.4664	8.717948	19.718	8.5752579	5.11598

 Table 1. Group-velocity spread Δv_{group} for the black hole Sgr A' and M31.

increases with respect to α_* and decreases with respect to β_* . The table H31 shows some values of Δv_{group} observable for the Sgr A' and M31 black holes.

Let us consider a special case called *blazar*, a jet which is associated with a relativistic outflow. It is emitted at a high Lorentz factor $\gamma_{\text{jet}} \gg 1$, also all regions throughout the close vicinity of a massive object (supermassive) around the BH. The light gets directed due to gravitational bending and appears from a young loose collimated place. In the case of the source, lens and observer are all in perfect alignment the light appears as a ring. We obtain the formulae for a blazar lens pattern slightly different from (3.12). Solving the Eq. (3.12) for γ_{jet} we have got

$$r_{\text{jet}}^2 = \frac{D_{\text{jet}}}{g(r_*)^2 \Gamma_{\text{jet}}^2} r_*^2, \quad (3.13)$$

where r_{jet}^2 is the position of the jet emission ring. Considering the non-relativistic limit $\gamma_{\text{jet}} \rightarrow 1$, the separation between source and observer is $D_{\text{jet}} = D_*$ and considering $D_{\text{jet}} \rightarrow \infty$ from Eq. H32 we find that

$$r_{\text{jet}}^2 = \frac{b_{\text{jet}}}{\Gamma_{\text{jet}}^2} (1 - \alpha_*) \quad (3.14)$$

from the explicit value of the α_* relativistic velocity $\beta_{\text{jet}} = \alpha_*$ and dropping the term $\gamma_{\text{jet}} \approx \gamma_{\text{jet}}^2$ and Γ_{jet}^2 in Eq. (3.14).

Eq. (3.14) shows that the radius decreases with an increase in α_* for both black holes. So, the maximum radius of blazar lens pattern is found dependent on the β_{jet} parameter.

B.3 Time delay

In this section we study the time delay between different images of a source black hole. The time delay comes due to the different paths and phases time while reaching the black hole. Here we follow the method discussed by the author Alvarado [34] to study time delay. The time taken by photons to reach an observer at infinity from the source is given by [34]

$$T(r) = \log \frac{b}{r} + \frac{2\pi b}{\alpha} \left(\frac{b}{r} - 1 \right) \quad (3.15)$$

where

$$f = \frac{1}{2\pi} \frac{d\theta}{dt}$$

is the

$$f = \frac{1}{2\pi} \frac{d\theta}{dt}$$

is the

where

$$f = \frac{1}{2\pi} \frac{d\theta}{dt}$$

is

where f is the frequency of the oscillation, θ is the phase angle, and t is the time. The frequency f is the number of cycles per second, and the phase angle θ is the angle between the reference signal and the signal being measured.

where

is

where f is the frequency of the oscillation, θ is the phase angle, and t is the time. The frequency f is the number of cycles per second, and the phase angle θ is the angle between the reference signal and the signal being measured.

The result of the calculation is

2.1. Period of the oscillation

The period of the oscillation is the time interval between two consecutive peaks of the signal. It is denoted by T and is measured in seconds. The period is the reciprocal of the frequency, i.e., $T = 1/f$.

where T is the period of the oscillation, f is the frequency, and θ is the phase angle.

is

where f is the frequency of the oscillation, θ is the phase angle, and t is the time. The frequency f is the number of cycles per second, and the phase angle θ is the angle between the reference signal and the signal being measured.

is

is

where f is the frequency of the oscillation, θ is the phase angle, and t is the time. The frequency f is the number of cycles per second, and the phase angle θ is the angle between the reference signal and the signal being measured.

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- [14] A. Kline, G. Pappas, *Phys. Rev. D* **67**(2003) 035001, [hep-th/0206166](#), T.W. Soudan, S.J. Clark, *AdS/CFT*, *Phys. Lett. B* **540**(2002) 1, [hep-th/0105152](#), K. Ghorbani, *Phys. Lett. B* **549**(2002) 146, [hep-th/0105171](#).
- [15] V.A. Kazakov, M. Mamon, *Phys. Rev. D* **44**(1991) 2666-69.
- [16] R. P. Kerr, *Phys. Rev. Lett.* **11** (1973) 237.
- [17] L. Teitelbaum, *Int. J. Number Theory* **15**(1993) 1.
- [18] L. M. Finkel, *Phys. Rev.* **48**, 1993 (1993).
- [19] P. Young, *Phys. Rev. D* **71**, 1277 (2005).
- [20] M. Goodman, M. H. Taroni and A. Todorok, *ArXiv:hep-th/9707042*.
- [21] A. Sen, *Phys. Rev. Lett.* **89**(1997) 119701.
- [22] K. Horiuchi and K. Murata, *Phys. Rev. D* **66**, 025003 (2002).
- [23] H. Hock, *Commun. Math. Phys.* **44**(1974) 149.
- [24] S. W. Kim and C. H. Kim, *Int. J. Theoretical Computer Science* **342** (2015).
- [25] U. Papen, E. Aghajanyan, G. Ghorbani, B. Aghajanyan, *Phys. Rev. D* **90**, 024003 (2014).
- [26] A. Aghajanyan, M. Aghajanyan, B. Aghajanyan, G. Ghorbani, *Phys. Rev. D* **93** (2016) 124043.
- [27] V. A. Kazakov, *Int. J. Number Theory* **10** (2014) 1119.
- [28] V. A. Kazakov and A. Sen, *Phys. Rev. D* **48**, 1994 (1993).
- [29] D. Gaiotto and V. A. Kazakov, *Phys. Rev. D* **93**, 125016 (2016).
- [30] V. A. Kazakov and M. Pospelov, *Phys. Lett. B* **343** (1995) 449.
- [31] D. Gaiotto and V. A. Kazakov, *Phys. Rev. Lett.* **95**, 141601 (2005).
- [32] D. Gaiotto and V. A. Kazakov, *Phys. Rev. Lett.* **94**, 081601 (2005).
- [33] K. Ghorbani and S. J. Clark, *Phys. Rev. D* **78** (2008) 125008.
- [34] R. Menezes, A. Sen and S. W. Kim, *Phys. Rev. D* **79** (2009) 125001 (2009).
- [35] S. W. Kim and S. J. Clark, *Phys. Rev. Lett.* **94**(2005) 021601.
- [36] H. J. de Vega, *Phys. Rev. D* **34** (1986).
- [37] E. Iizuka, *Advances in Quantum Class. Quantum Grav.* **30** (2013) 134011.
- [38] A. Sen, *Phys. Rev. Lett.* **93**, 231601 (2004) [hep-th/0402053](#).
- [39] A. H. Jubin, *Int. J. Geom. Methods Mod. Phys.* **14**, 1750023 (2017).
- [40] R. Menezes, *Commun. Math. Phys.* **271**(2007) 1149, [hep-th/0602412](#) (2006).
- [41] T. Mook, *Int. J. Mod. Phys. A* **24**(2009) 123.
- [42] A. Sen, *Class. Quantum Gravity* **21** (2004) R1.
- [43] A. Sen, *Int. J. Number Theory* **10** (2014) 1119.
- [44] A. Sen, *Int. J. Number Theory* **10** (2014) 1119.
- [45] S. W. Kim, *Int. J. Number Theory* **10** (2014) 1119.

- [64] K. Kuster and A. H. Mahdipour, *Class. Quant. Grav.* **25**, 175015 (2008).
- [65] M. A. Arzouman, F. A. Betea, A. V. Turpin, *Phys. Rev. D* **100**, 124004 (2019).
- [66] A. Arzouman, B. Alameddine, A. Abdelghany, *Phys. Rev. D* **100**, 084004 (2019).
- [67] R. G. J. van den Broek, *Phys. Lett. A* **333**, 388-395 (2015).
- [68] Shao-Meng Hu, Yu-Ming Lu, *Chin. Phys. Lett.* **34**, 041101 (2017).
- [69] A. V. Turpin, G. V. Toupas, *Phys. Rev. D* **100**, 104002 (2019).
- [70] A. Abdelghany, B. Alameddine, A. Hachem, A. Hachem, *Int. J. Mod. Phys. D* **27**, 1750051 (2017).
- [71] G. Z. Baber, A. Z. Baber, F. Alotaibi, *Phys. Rev. D* **100**, 084001 (2019).
- [72] B. Abdelghany, *Phys. Rev. D* **100**, 084007 (2019).
- [73] A. Abdelghany, M. Alotaibi, B. Alameddine, C. Ghosh, *Phys. Rev. D* **100**, 244004 (2019).
- [74] A. Li, Y. C. Zhang, J. Zhang and F. Chen, *Acta Phys. Sin.* **39**, 1137 (1984).
- [75] B. Alameddine and A. Abdelghany, *Phys. Rev. D* **100**, 084004 (2019).
- [76] Y. H. Wu, F. Chang, Y. Zhang and B. Y. Zhou, *Chin. Phys. Lett.* **38**, 041101 (2015).
- [77] L. Q. Hu, M. Lu, and Y. Han, *Phys. Rev. D* **100**, 044005 (2019).
- [78] A. A. Starinets, *Adv. High Energy Phys.* **2017**, 1 (2017).
- [79] A. A. Starinets, *Adv. High Energy Phys.* **2017**, 1 (2017).
- [80] B. Alameddine, H. Hachem, F. J. Figueroa, *Class. Quantum Grav.* **36**, 024004 (2019).
- [81] D. G. Mubshir, F. J. Figueroa, *Phys. Rev. Lett.* **113**, 021104 (2014).
- [82] D. G. Mubshir, B. L. Tsou, E. D. Perlmutter, *Phys. Rev. Lett.* **34**, 1377 (1975).
- [83] G. Baber, Mubshir-Kyriakidis, G. V. Toupas, *Mod. Phys. Lett. A* **33**, 175013 (2018).
- [84] K. Baber, A. Hachem, *Phys. Rev. D* **100**, 084001 (2019).
- [85] A. V. Turpin, *Phys. Rev. D* **100**, 084001 (2019).
- [86] F. Alotaibi, B. Alameddine, A. Abdelghany, *Phys. Rev. D* **100**, 084003 (2019).
- [87] A. Kagan, *Mod. Phys. Lett. A* **33**, 1750002 (2018).
- [88] G. Z. Baber, A. Z. Baber, F. Alotaibi, *Grav. Phys. Lett.* **16**, 010101 (2019).
- [89] G. V. Toupas and G. Z. Baber, *Grav. Phys. Lett. A* **33**, 1750002 (2018).
- [90] A. Abdelghany, B. Alameddine, A. Hachem, A. Hachem, *Int. Mod. Phys.* **34**, 1750001 (2017).
- [91] A. V. Turpin, *Grav. Phys. Lett. A* **33**, 1750002 (2018).
- [92] G. V. Toupas and G. Z. Baber, *Grav. Phys. Lett. A* **33**, 1750002 (2018).
- [93] G. V. Toupas and G. Z. Baber, *Grav. Phys. Lett. A* **33**, 1750002 (2018).
- [94] A. Abdelghany, B. Alameddine, and A. A. Tawfik, *Astronomy Space Sci.* **348**, 325 (2021).
- [95] B. Alameddine, *Grav. Phys. Lett. A* **33**, 1750002 (2018).
- [96] A. V. Turpin, *Grav. Phys. Lett. A* **33**, 1750002 (2018).

- [1] B. Z. Kuvshinov, Z. Naturforsch. Teil A, 67(1992) 2241-2244 Phys. Scripta 457 1015-1024
- [2] Information, Extraction of energy and charge from black-hole Physics, Phys. Rev. D7(1973) 144-151
- [3] R. H. Price, Gen. Rel. & Gravitation Astrophysical Journal 18(1986)1879
- [4] A. Teukolsky, Phys. Rev. Lett. 29 14 (1972)
- [5] S. A. Teukolsky, W. H. Press, Astrophys. J. 102 413 (1976)
- [6] L. Teuk, V. Chandrasekhar, P. Pain, Lecture Notes in Physics (Gravitation) 97 (1978)
- [7] A. Ashtekar, D. Christodoulou, D. Isenberg, V. Katugiri and M. Misner, Phys. Rev. 143(2):1175-1176
- [8] A. Ashtekar, S. Fairweather, Phys. Rev. 149(2):1041-1049
- [9] T. Tsuru, V. Chandrasekhar, G. G. Thorne, A. M. L. Lyth, Phys. Rev. Lett. 48(12) 1311-1314
- [10] L. Teuk, V. Chandrasekhar, P. Pain, Phys. Rev. Lett. 42(11):655-658
- [11] S. A. E. Henderson and P. Pain, Phys. Rev. 141 213 (1966) 211-212
- [12] V. Chandrasekhar, Q. J. Math. Phys. 21 199 (1970) 199-204
- [13] Y. B. Zaslavskii, H. S. Yoon, A. A. Grib, Class. Quantum Grav. 2, 245-252 (1984)
- [14] G. V. Kuvshinov, Class. Quantum Grav. 15(1998), 2419
- [15] L. V. Teuk, Phys. Scripta 31, 36 (1975)
- [16] Euro. Phys. J. 16(1997) 2019
- [17] M. Kuvshinov, Phys. Rev. 14(1998) 2404 (1998)
- [18] M. Kuvshinov, M. Kuvshinov, Phys. Rev. 14(1998) 2404 (1998)
- [19] M. Kuvshinov, A. Teukolsky, H. Price, Phys. Rev. 14(1998)
- [20] S. Teuk, Phys. Lett. B 70(1977) 105 (1977) 105-107
- [21] E. Abeyaratne, Astrophys. J. 347 2075
- [22] E. Abeyaratne, Astrophys. J. 347 2075
- [23] E. Abeyaratne, Astrophys. J. 347 2075
- [24] E. Abeyaratne, Astrophys. J. 347 2075
- [25] E. Abeyaratne, Astrophys. J. 347 2075
- [26] E. Abeyaratne, Astrophys. J. 347 2075
- [27] E. Abeyaratne, Astrophys. J. 347 2075
- [28] T. Tsuru, H. S. Yoon, D. Christodoulou, Phys. Scripta 457 1015-1024
- [29] E. Grib, D. Christodoulou, D. Isenberg, E. Katugiri, S. Misner, R. Misner, Astrophys. J. 143(2):1175-1176
- [30] E. Abeyaratne, Astrophys. J. 347 2075
- [31] E. Abeyaratne, Astrophys. J. 347 2075
- [32] E. Abeyaratne, Astrophys. J. 347 2075

- [illegible]

[51] M. Sedrakian, *Ann. Rev. Math. Sci. Eng.* **30**, 17

[52] M. H. Hahner, R. Howard, P. Padgett, and B. Pomeroy, *Phys. Rev. Lett.* **67**, 1241 (1991)

[53] B. A. Auluck, *Proc. R. Soc. London, Ser. A* **240**, 361

[54] A. Kuznetsov and P. E. Sulem, *Journal of Mathematical Physics* **34**, 446 (1993)

[55] J. D. DeGroot and S. Yip, *Advances in Physics* **19**, 1 (1980)

[56] B. Auluck, *Phys. Rev.* **48**, 884 (1942)

[57] G. G. Thomson, V. G. Tsinosider, *Phys. Rev.* **44**, 747 (1947)

[58] A. A. Kuznetsov and M. Hahner, *Phys. Rev.* **44**, 1510 (1991)

[59] A. A. Kuznetsov and M. Hahner, *Phys. Rev.* **44**, 1508 (1991)

[60] A. A. Kuznetsov, *Phys. Rev.* **48**, 6964 (1993)

[61] P. E. Sulem and A. Kuznetsov, *Phys. Rev.* **45**, 4761 (1992)

[62] L. Kuznetsov, P. A. Kuznetsov, *Phys. Rev. Lett.* **70**, 1174 (1993)

[63] A. A. Kuznetsov, A. A. Kuznetsov, and B. Pomeroy, *Phys. Rev. Lett.* **71**, 2111 (1993)

[64] J. D. DeGroot, *Phys. Rev. Lett.* **67**, 1241 (1991)

[65] A. A. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[66] A. A. Kuznetsov, *Phys. Rev.* **48**, 6964 (1993)

[67] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[68] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[69] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[70] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[71] A. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[72] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[73] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[74] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[75] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[76] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[77] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[78] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[79] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[80] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[81] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[82] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[83] L. Kuznetsov, *Phys. Rev. Lett.* **71**, 2111 (1993)

[104] J. von Neumann, *Math. Ann.* 102 (1929).

[105] J. von Neumann, *Math. Ann.* 102 (1929).

[106] W. K. Hastings, *Phys. Rev. Lett.* 45 (1980).

[107] K. G. Wilson, *Phys. Rev. D* 10 (1974).

[108] K. G. Wilson, *Phys. Rev. D* 10 (1974).

[109] E. Lieb, *Phys. Rev. Lett.* 22 (1969).

[110] J. von Neumann, *Math. Ann.* 102 (1929).

[111] J. von Neumann, *Math. Ann.* 102 (1929).

[112] J. von Neumann, *Math. Ann.* 102 (1929).

[113] J. von Neumann, *Math. Ann.* 102 (1929).

[114] J. von Neumann, *Math. Ann.* 102 (1929).

[115] J. von Neumann, *Math. Ann.* 102 (1929).

[116] J. von Neumann, *Math. Ann.* 102 (1929).

[117] J. von Neumann, *Math. Ann.* 102 (1929).

[118] J. von Neumann, *Math. Ann.* 102 (1929).

[119] J. von Neumann, *Math. Ann.* 102 (1929).

[120] J. von Neumann, *Math. Ann.* 102 (1929).

[121] J. von Neumann, *Math. Ann.* 102 (1929).

[122] J. von Neumann, *Math. Ann.* 102 (1929).

[123] J. von Neumann, *Math. Ann.* 102 (1929).

[124] J. von Neumann, *Math. Ann.* 102 (1929).

[125] J. von Neumann, *Math. Ann.* 102 (1929).

[126] J. von Neumann, *Math. Ann.* 102 (1929).

[127] J. von Neumann, *Math. Ann.* 102 (1929).

[128] J. von Neumann, *Math. Ann.* 102 (1929).

[129] J. von Neumann, *Math. Ann.* 102 (1929).

[130] J. von Neumann, *Math. Ann.* 102 (1929).

[131] J. von Neumann, *Math. Ann.* 102 (1929).

[132] J. von Neumann, *Math. Ann.* 102 (1929).

[133] J. von Neumann, *Math. Ann.* 102 (1929).

[134] J. von Neumann, *Math. Ann.* 102 (1929).

[135] J. von Neumann, *Math. Ann.* 102 (1929).

[64] L. S. Buzdakov, *Regge and O⁺ Regge*, Moscow (1972).

[65] A. A. Abrikosov, B. I. Lazduski, *Strong L. Ward's Identities*, *Int. J. Mod. Phys.* **1**(1986) 1749-1756.

[66] A. A. Abrikosov, B. I. Lazduski, N. I. Medvedev, *Quantization*, *Phys. Rev. Lett.* **56**(1976) 2049.

[67] E. A. Amaldi, *Quarks*, P. A. Mandelstam, A. Martin, *Phys. Rev.* **178**(1969) 1484.

[68] Y. Kuroki, O. Tegen, *Regge*, *Phys. Rep.* **147**(1987) 1819.

[69] P. V. P. Cunha, C. A. B. Penedes, *Regge*, *Phys. Rev. Lett.* **121**(2018) 191601.

[70] M. B. Green, B. I. Lazduski, *Regge*, *Phys. Rev. Lett.* **124**(2020) 191601.

[71] B. I. Lazduski, M. B. Green, *Regge*, *Phys. Rep.* **77**(1979) 109.

[72] Hong G. Yang, *The Yang-Mills Equations and Quantum Chromodynamics*, *Theor. Phys.* **16**(1975) 172.

[73] A. Regge, *Phys. Rev.* **191**(1973) 570.

[74] M. B. Green, *Phys. Rep.* **167**(1988) 187.

[75] J. A. O'Connell, *Regge*, *Phys. Rep.* **182**(1990) 159.

[76] B. I. Lazduski, L. S. Buzdakov, A. Martin, *Phys. Rev. Lett.* **56**(1976) 2049.

[77] E. Amaldi, P. A. Mandelstam, B. I. Lazduski, *Phys. Rev. Lett.* **27**(1971) 1712.

[78] M. B. Green, A. A. Abrikosov, P. A. Mandelstam, *Phys. Rev. Lett.* **36**(1976) 1304.

[79] A. A. Abrikosov, B. I. Lazduski, *Regge*, *Phys. Rev. Lett.* **36**(1976) 1304.

[80] I. P. Ivanov, *Regge*, *Phys. Rep.* **159**(1988) 1.

[81] A. O'Connell, *Regge*, *Phys. Rep.* **182**(1990) 159.

[82] B. I. Lazduski, *Regge*, *Phys. Rep.* **167**(1988) 187.

[83] A. Koma, *Regge*, *Phys. Rep.* **182**(1990) 159.

[84] A. A. Abrikosov, B. I. Lazduski, A. A. Mandelstam, *Phys. Rev. Lett.* **36**(1976) 1304.

[85] W. G. Yang, *Regge*, *Phys. Rep.* **182**(1990) 159.

[86] A. O'Connell, *Regge*, *Phys. Rep.* **182**(1990) 159.

[87] B. I. Lazduski, *Regge*, *Phys. Rep.* **167**(1988) 187.

[88] M. B. Green, *Regge*, *Phys. Rep.* **182**(1990) 159.

[89] E. A. Amaldi, *Regge*, *Phys. Rev. Lett.* **27**(1971) 1712.

[90] A. A. Abrikosov, B. I. Lazduski, *Regge*, *Phys. Rev. Lett.* **36**(1976) 1304.

[91] A. O'Connell, *Regge*, *Phys. Rep.* **182**(1990) 159.

[92] A. A. Abrikosov, B. I. Lazduski, *Regge*, *Phys. Rev. Lett.* **36**(1976) 1304.

[93] B. I. Lazduski, *Regge*, *Phys. Rep.* **167**(1988) 187.

[94] M. B. Green, *Regge*, *Phys. Rep.* **182**(1990) 159.

- [147] G. Adkins and R. Coquard, *Advanced Graduate Geophysics: Elastic & Physical Properties of Rocks*, Elsevier, 1981.
- [148] F. Bouchaud and Y. Lemaire, *Phys. Rev. Lett.* **68**, 3111 (1992).
- [149] L. Bouchaud and F. Tarantola, *IBP 65* 141 (2004).
- [150] A. Constantin and F. Ernst, *Phys. Rev. Lett.* **69**, 3463 (1992).
- [151] T. Schmalz and G. Zürn, *Phys. Rev. Lett.* **112**, 251601 (2014).
- [152] G. Antoniadou, A. Baskopoulou, F. Kallin, *Phys. Rev. Lett.* **107**, 020504 (2011).
- [153] L. Antoniadou, A. Baskopoulou, F. Kallin, *Phys. Rev. Lett.* **107**, 090601 (2011).
- [154] M. Hertz and U. Gansel, *Phys. Rev. Lett.* **90**, 045501 (2003).
- [155] F. Bouchaud, Y. Lemaire, G. Adkins, R. Coquard, F. Antoniadou, *Phys. Rev. Lett.* **94**, 054301 (2005).
- [156] F. Bouchaud, Y. Lemaire, G. Adkins, F. Antoniadou, *Geophysical Research Letters* **31**, L17307 (2004).
- [157] F. Bouchaud, R. Coquard and G. Adkins, *Phys. Rev. Lett.* **90**, 145501 (2003).
- [158] T. F. Schmalz, *Comm. Comput. Geom.* **32**, 114001 (2017).
- [159] G. Adkins, R. Coquard, E. de la Roche, A. Baskopoulou, Z. Schmalz, *Phys. Rev. Lett.* **109**, 020504 (2012).
- [160] A. Constantin, *Phys. Rev. Lett.* **69**, 1846 (1992).
- [161] Y. Lemaire, *Phys. Rev. Lett.* **68**, 1846 (1992).
- [162] Y. Lemaire, G. Adkins, R. Coquard, F. Antoniadou, *Phys. Rev. Lett.* **94**, 154301 (2005).
- [163] Y. Lemaire and G. Adkins, *Phys. Rev. Lett.* **94**, 154304 (2005).
- [164] L. W. Anand, and M. E. Warren, *Ann. Geophys.* **35**, 273 (1992).
- [165] F. Bouchaud and G. Tarantola, *Phys. Rev. Lett.* **71**, 2344 (1993).
- [166] *Geophysical Computations and Computer Programming and Applications of the General Theory of Elasticity* (Elsevier, 1977).
- [167] S. G. Ghosh, L. Kaur, and S. U. Islam, *Comput. Math. Appl.* **61**, 405 (2011).
- [168] A. Adhikari, Y. Yamada, T. Omi, T. Katsuragi, H. Kato, *Phys. Rev. Lett.* **114**, 015501 (2015).
- [169] M. G. Koss, *Science, Technological Computing, and the Social and Natural Sciences* (Prentice-Hall, New Jersey, 1978).
- [170] M. Warren, *Phys. Rev. Lett.* **66**, 942 (1991).
- [171] T. Omi, *Ann. Phys.* **265**, 44 (2001).
- [172] T. Omi, A. Adhikari and H. Kato, *Phys. Rev. Lett.* **114**, 015501 (2015).
- [173] G. Constantin, G. Adkins and R. Coquard, *Phys. Rev. Lett.* **69**, 1846 (1992).
- [174] M. Adkins, L. Kaur, S. G. Ghosh, *Sci. Data* **5**, 20127 (2017).
- [175] R. Coquard, S. G. Ghosh, *Comm. Comput. Geom.* **14**, 000018 (2018).
- [176] R. Coquard, M. Anand, A. Baskopoulou, M. G. Koss, F. Bouchaud, *Commun. Comput. Geom.* **11**, 000011 (2017).
- [177] A. Adhikari, M. Katsuragi, S. Yamada and S. F. Miao, *Vibrationally changed rock failure under compression* (4th International Conference on Geotechnical Engineering, Springer, Singapore, Singapore, 2019).

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Quantitative quality and particle masses in 3D and nonconformal spherical structures

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The Inflationary Scenario in the Λ CDM Gravity Model with a R^2 Term

LI, J. AND LIU, Y. (2019) THE INFLATIONARY SCENARIO IN THE Λ CDM GRAVITY MODEL WITH A R^2 TERM. *MONIST*, 100(1), 1-10.

ABSTRACT. In this paper, we study the inflationary scenario in the Λ CDM gravity model with a R^2 term. We find that the inflationary scenario is possible in this model. The inflationary potential is derived, and the inflationary parameters are calculated. The results show that the inflationary scenario is possible in this model.

KEYWORDS: Λ CDM, inflation, R^2 term, gravity model

1. INTRODUCTION

INFLATION is a period of rapid expansion in the early universe, which is necessary to explain the observed homogeneity and isotropy of the universe.

The inflationary scenario is based on the assumption that the universe is dominated by a scalar field, called the inflaton. The inflaton field is responsible for the rapid expansion of the universe during the inflationary period.

In this paper, we study the inflationary scenario in the Λ CDM gravity model with a R^2 term. We find that the inflationary scenario is possible in this model.

2. THE Λ CDM GRAVITY MODEL WITH A R^2 TERM

The Λ CDM gravity model with a R^2 term is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + \int d^4x \sqrt{-g} \Lambda,$$

where R is the Ricci scalar, g is the determinant of the metric tensor, ϕ is the inflaton field, and $V(\phi)$ is the potential of the inflaton field.

The equation of motion for the inflaton field is given by

$$\Box \phi = -\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \phi} = -\frac{1}{\sqrt{-g}} \frac{\delta}{\delta \phi} \left(\int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + \int d^4x \sqrt{-g} \Lambda \right).$$

3. INFLATIONARY SCENARIO

In this section, we study the inflationary scenario in the Λ CDM gravity model with a R^2 term.

1. **Introduction**
 2. **Methodology**
 3. **Results**
 4. **Discussion**
 5. **Conclusion**
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Figure 1 *Flowchart of the study*

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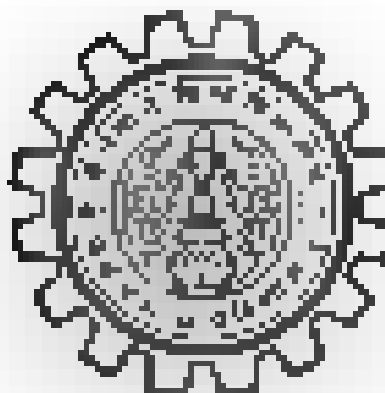
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Background

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Study of black holes in standard and modified gravity

**Thesis submitted for the award of
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Submitted by

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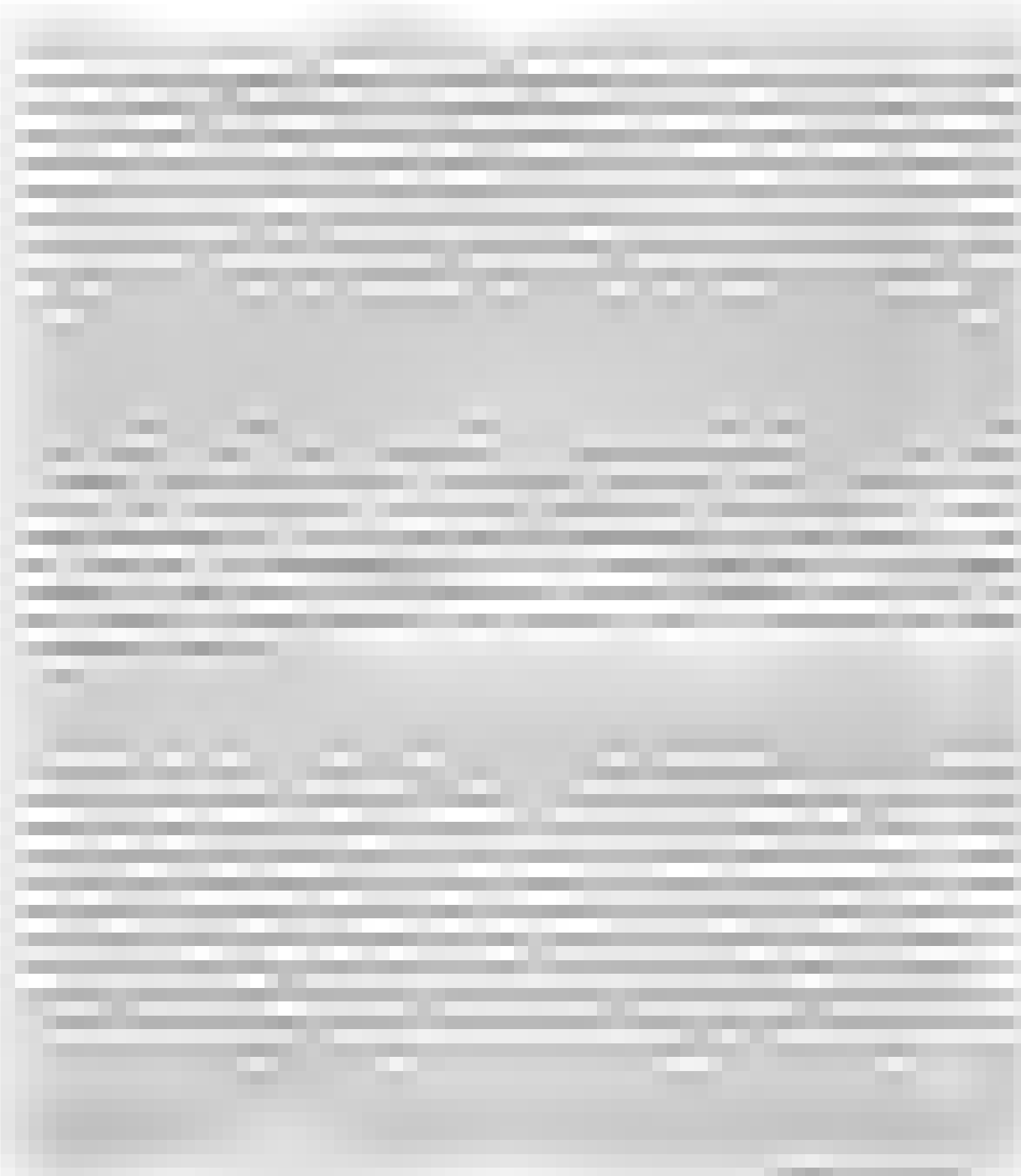
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Hareghy Mahalan College, Chinsurah, Hareghy

And

Durgapukur Govt. College, Durgapukur, Burdwan

Chapter 9



appears because of the use of non-commutative spacetime proposed by Snyder. The interfering term that appear are very very small since the term contains the product of two very small factors λ , and l . Although the interfering term is negligibly small, it has some theoretical consequences, e.g., symmetric contribution may get disturbed by an asymmetric term - the reverse perhaps does not get so. It is true that both the parameters λ , and l are small. A physical result containing some arbitrary parameter would be physically sensitive if the parameters can be constrained from experimental observation. Both the parameters can be put under constraints using the same arguments available in [65] and [64]. The conjecture used there is that the correction would be of the order of general relativistic corrections. Here, we can, indeed, put a restriction on the parameters λ , and l following the same argument. The upper bound of the predicted values of the parameters is given in Table (2.1). Note that the predicted values of l are identical to the values obtained in the article [64]. It is indeed the expected result. Our study enables us to put another constraint on the parameter λ , when the constraint on l is known if it is taken into consideration that the correction that appeared in the form of interference which is proportional to λ, l would be of the same order of magnitude of the pure terms. Note that the first-order correction term is proportional to λ, l . So if we are intended to constrain the interfering term, it will provide new information that the parameters may have reciprocal constraint between each other. When the contribution through one parameter dominates, the contribution from the other will get suppressed automatically.

Chapter III contains a description to establish that Kerr-Sen-like spacetime is a viable solution of Einstein-bumblebee gravity model [74] where LV scenario gets involved through the said bumblebee field vector B_μ . Then, it has been extended to study the properties of the shadow it is supposed to cast. To this end, the effective potential that results from the null geodesics in the bumblebee rotating black hole spacetime is computed that in turn helps to get the tentative nature of the motion of photon predicted for different choices of the parameters a , b , and l . Then the nature of shadow is studied and how does the shadow gets deformed has also been studied for different variations of a , b , and l . We observe that the shadow gets shifted towards the right for positive l and shifted towards left for negative l when a and b remain fixed. However, shifting is always towards the right when b increases whatever the set of values of l and a are taken. We have also studied the rate of emission of energy for this type of black hole. The emission rate decreases when b increases for any set of fixed values of a and l . It also decreases when l increases for an arbitrary set of fixed values of a and b . A crucial difference however in the situation is noticed when l increases. The emission rate although decreases with the increase of l like the case when b increases, the peak of the curve gets shifted towards lower a in the situation when l increases. The deformation of the shadow for the Kerr-Sen-like black hole in the Einstein-bumblebee gravity model with the variation of l observed here is a theoretical prediction. It shows that it enhances the distortion of shadow, and it would be detected by the new generation gravitational antennas.

Chapter IV deals largely with the intermediate effect on the Kerr-Sen-like spacetime which has been found as a solution to the equations developed from Einstein-bumblebee gravity in [74] which has a LV parameter l . This modified spacetime is used to study the propagation of light in a non-magnetized pressure-less plasma in [115]. We have considered the plasma as a medium with dispersive properties given by a frequency-dependent index of refraction. The gravitational field is determined by the mass, the spin, and the bumblebee parameter. The Gravitational field due to plasma is neglected here. We have studied how the nature of shadows gets affected by the LV effect associated with the bumblebee field in the pressure-less nonmagnetic plasma. We have also studied the energy emission scenario and the weak field lensing in the Kerr-Sen-like spacetimes due to Einstein-bumblebee gravity background when it is veiled in a plasma medium. The results for Kerr, Kerr-like, and Schwarzschild black holes can, as well, be obtained from this result with a suitable limit. We have investigated in detail the impact of the charge ($Q^2 = 4$), plasma parameter k , and the Lorentz violating bumblebee parameter l on the structure of shadow, on the lensing effect of light, and on the energy emission due to black hole radiation. It has been found that the new parameters quantitatively influence the structure of the event horizon by altering the radius significantly. It is observed that the size of the shadow viewed by a distant observer reduces with an increase in the charge Q and the size of the shadow appears to be larger with the increase in the plasma parameter k . Therefore, the

hambhalee field maintains its action of deforming the shadow in presence of plasma as well. Since the energy liberated from the black hole depends on the area of the shadow, the rate of energy emission from the black hole is higher when the black hole is surrounded by plasma. As far as the angle of deflection is concerned, the photons are observed to experience an increase in the deviation as the plasma factor k increases. While on the other hand, the angle of deflection reduces sufficiently when the amount of charge parameter increases. It is also observed that the deflection angle increases with an increase in the value of the IV parameter l . Sufficient negative l may cause the lensing effect too!

Chapter V is devoted to constraining the free parameters that have entered into the modified models adopted here. The idea behind this is as follows. The theory of general relativity predicts that the Kerr metric is capable of describing astrophysical black holes. However, such black holes may not exist in a perfect vacuum due to the presence of surrounding matter, like plasma, dark matter, etc. The accretion disks may also alter the black hole, which may not fit with the Kerr metric. The hambhalee-inspired Kerr spacetime enables the incorporation of potential deviations from the Kerr metric through the additional deviation parameters. The advantage of this type of modified gravity is that it is a generic one and it has the ability to cover the prediction of the Kerr metric since the introduced parameters can be chosen in such a way that it makes the null deviation of the shadow from Kerr spacetime. There is a theoretical motivation for modified theories of gravity indeed however the constraining of the free parameter from the experimental observation allows us to examine the viability of the model. In this context the information of the captured image by the EHT collaboration become instrumental.

The EHT collaboration has captured the image of supermassive black hole M87* exhibiting a deviation from circularity $\Delta C \leq 0.10$. The observation did not say anything about modified theories of gravity or suggest any alternatives to the Kerr black hole. Motivated by this, we considered Kerr-Sen-like modified black holes, which have additional deviation parameters l and k to the Kerr black hole. The presence of Plasma adds an additional parameter. We use the deviation from circularity $\Delta C \leq 0.10$ as deduced by the EHT collaboration to constrain the Kerr-Sen like black hole parameters. We have observed that the bound $\Delta C \leq 0.10$ is satisfied by black hole shadows over a finite parameter space both in the presence of plasma and in the absence of it. Besides, the information available from the article [117] allows us to put constrain on the value of l . We have found that the value of the parameter $l > -1$ and the upper bound of it is 2.615291. So l can take both positive and negative values. We would like to mention, that in the calculation of the rate of energy emission, we did not consider the greybody factor. The mathematical formulation of it is much involved for any black hole. Moreover, it is important when quantitative estimation is brought under consideration. Unfortunately, there is no available data to compare it with the real situation. A fascinating and involved extension of this work would be to evaluate it theoretically.

We have described the superradiance phenomena of the scalar field scattered off Kerr-Sen-like black holes in detail in Chapter VI. The contents of this chapter have been studied in [126]. We have already mentioned that this is a black hole solution to Einstein-hambhalee gravity model. A Lorentz violating parameter is there in the spacetime metric of this modified gravity. Here, Lorentz violation takes place via a spontaneous symmetry breaking when the pseudovector field of the Einstein-hambhalee model receives a vacuum expectation value. We have also studied superradiance instability. We have studied here how the superradiance phenomena and related instability scenario gets influenced by the Lorentz violation effect through the Kerr-Sen-like background which is the outcome of the Einstein-hambhalee gravity model. Employing asymptotic matching of the scalar wave, it has been established that in the low-frequency limit, i.e., for $\omega < \omega_{\text{crit}}$, the scalar waves become superradiant amplified. The numerical computation, however, shows that for $m < 0$ the massive scalar field has a non-superradiant mode. For $m \geq 0$ it is superradiant. The superradiant process enhances with the decrease of the Lorentz violation parameter and the reverse is the case when the Lorentz violation parameter increases irrespective of the sign of the value of this parameter. We have also demonstrated here the superradiant process gets influenced by the parameter k . From Fig. (6.10) it is clear that superradiance enhances with the decrease of the positive value of the parameter k . Note that $k = \frac{q^2}{2}$. So it cannot be negative. Moreover, extending the issue of the black hole back, the analytical study of superradiant instability is made. The plots in Fig. (6.11) related to the study reveal

that the LV parameter remarkably affects the instability region. In the background with the negative LV parameters, the scalar field has more chances to acquire unstable dynamics and for the positive values of the LV parameter, it is less. Therefore, the Lorentz violation has a significant influence on the superradiance scattering phenomena and the corresponding instability linked with it.

As an extension, we describe in Chapter VII how a framework is developed where quantum correction due to the Lorentz violation and non-commutative character of spacetime has been taken into account on the same footing [121]. The spacetime background involves a non-commutative Kerr-like Lorentz symmetry violating black hole. We have extensively studied different aspects of the non-commutative and Lorentz violating Kerr-like black hole. The spin, the mass, the Lorentz violation parameter, and the non-commutative parameter involved in it determines the gravitational field of this black hole. First of all, we study geometry in detail concerning its horizon structure and ergosphere. The article [121] contains the study of two important optical phenomena in the vicinity of this black hole. In this respect, we have considered the superradiance phenomena and find that it crucially depends on the parameter l and θ_0 , apart from its dependence on a which is linked with the spin of the black hole. The superradiance process enhances with the decrease in the value of the Lorentz violation parameter and it diminishes when we increase the value of the Lorentz violation parameter. We also observe that with the increase in the value of the parameter θ_0 , the superradiance process gets diminished. However, with the increase in the value of a the superradiance process increases. Next, we have brought into our investigation the effect of Lorentz violating parameter l and non-commutative parameter θ_0 on the size of the black hole shadow. We have observed that the size of the black hole shadow increases with an increase in the value of the parameter l and it decreases with an increase in the value of the parameter θ_0 . Thus, it can be safely concluded that Lorentz violation and non-commutativity, both, have significant impacts on black hole shadow. We have also studied energy emission rates. These results have clearly established the influence of the Lorentz violation parameter and non-commutative parameter on emission rate. We observe that the energy emission rate decreases with an increase in the value of b for any set of fixed values of a and l . It also decreases with an increase in the value of l , when a and b are held fixed, and with an increase in the value of a , if l and b are kept fixed.

We have been able to constrain the parameters even in the modified theories augmented with the combined effect of Lorentz violation and the non-commutative character of spacetime using the observations of EHT collaboration. For inclination angle $\theta = 30^\circ$, the deviation from circularity $\Delta C \leq 0.1$ and angular diameter $\theta_d = 42 \pm 3 \mu\text{as}$ within 1σ region are satisfied for finite parameter space $(\frac{\hat{a}}{M} - \frac{\hat{a}}{M})$. For inclination angle $\theta = 17^\circ$, the circularity deviation $\Delta C \leq 0.1$ is satisfied for the entire parameter space $(\frac{\hat{a}}{M} - \frac{\hat{a}}{M})$. The angular diameter $\theta_d = 42 \pm 3 \mu\text{as}$ within 1σ is satisfied for finite parameter space $(\frac{\hat{a}}{M} - \frac{\hat{a}}{M})$. The axis ratio D_d satisfies the constraint $1 < D_d \leq 4/3$ for the entire parameter space at both the inclination angles $\theta = 30^\circ$ as well as $\theta = 17^\circ$. Therefore our study enables us to establish the fact that non-commutative Kerr-like black holes are remarkably consistent with EHT images of M87. It demands that ruling out non-commutative Kerr-like black holes from the observational data of black hole shadow would be illogical. Thus, non-commutative Kerr-like black hole may be considered a viable candidate for the astrophysical black hole. It has also been shown that the possible upper bound of θ_0 , which is associated with the non-commutativity, is 0.00000073 M^2 . It is intriguing and indeed a novel way to constrain the parameter associated with the non-commutativity from the shadow of an astronomical black hole. Till now we do not have any available data to constrain the parameter l and θ_0 from the superradiance effect and from the energy emission process. So, the parameter cannot be constrained by knowledge of the superradiance phenomena and the energy emission process. We have made all the plots for the superradiance and the energy emission process maintaining the constraint obtained from the EHT data concerning the shadow of the M87.

Chapter VIII is devoted to study the strong as well as weak lensing due to the black hole modified with scalar hair. Black holes have been considered more than two decades since the Einstein equations for quite some time now. Nevertheless, it is only recently that their direct existence was detected substantially due to the report of the results of the Event Horizon Telescope Collaboration. So, the study of aspects of black holes and the consequences of their presence have now surfaced with new alleviation. In this environment, we study strong as well as weak lensing in the background of a

modified black hole. Hence, modification due to scalar hair is considered in [21] and an attempt has been made to study breaching mechanism and how the presence of hair influence the breaching property. The rationale adopted here evokes the no hair theorem since the hair is assumed to be restricted by finite source from the surroundings, such as dark matter, which is considered to have a constant energy-momentum tensor. This modification pertained to a uniform parameter α_0 and β_0 . The formation of black hole is the determination because of the presence of outside supply and the β_0 is associated with alteration in thermodynamical process because of the presence of hair. We observe that the breaching coefficient β increases with α_0 but the breaching coefficient $\tilde{\beta}$ and the impact parameter b_{ph} decrease with the increase of α_0 . However, with the increase of α_0 although α decrease $\tilde{\beta}$ goes unchanged. The impact parameter b_{ph} increases with the increase of β_0 . The value of the parameter resembles Schwarzschild black hole whenever we put $\alpha_0 = 0$.

The modified results that we have presented in the past have their own advantages in order to understand supermassive black holes through the reasoning that the observations of the EHT only information has spread recently. However, it is still too early to say which model fits the experimental data or performs better than another model. As a result, numerous investigations in this regard need to be pursued. While conducting our research, we focused on the information pertaining to [19]. What further insight does the information from [22] provide about these modifications that we will bring next up?